LECTURE 1

- **Readings:** Sections 1.1, 1.2

Lecture outline

- General course information
- Sample space of an experiment
  - Examples
- Axioms of probability
  - More examples

Course Information

- 6.041/6.431 — Probability
  - Lecturer: Muriel Médard,

- Pick up **and read** syllabus
- **Turn in section assignment form**

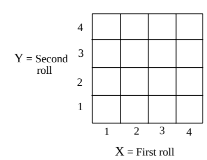
- Pick up copy of slides, homework #1 (due 2/9)
- Text: Intro to Probability (Bertsekas/Tsitsiklis)
- Grading
  - Class-time quizzes: Quiz 1 3/7, 12-1pm (22%)
  - Quiz 2: 4/11, 12-1pm (28%)
  - Final exam (35%)
  - Weekly homework (10%)
  - Interest/effort/mastery (5%)
- Collaboration policy: in syllabus
- Subject: modelling and analysis of experiments with uncertain outcomes
- Applications: engineering, management, and some frivolous

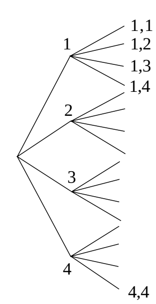
Sample space

- List of possible outcomes
- List must be:
  - Mutually exclusive
  - Collectively exhaustive
  - At the "right" granularity

Sample space examples

- Two rolls of a tetrahedral die
  - Sample space vs. sequential description





- A continuous sample space: (x, y) such that $0 \leq x, y \leq 1$

Axioms of probability

- **Event:** a subset of the sample space
- Probability is assigned to events

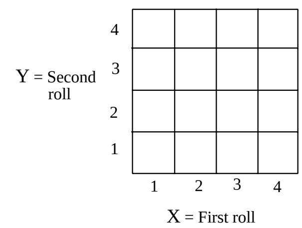
Axioms:

1. $\mathbf{P}(A) \geq 0$
2. $\mathbf{P}(\text{universe}) = 1$
3. If $A \cap B = \emptyset$, then $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B)$

- $\mathbf{P}(\{s_1, s_2, \ldots, s_k\}) = \mathbf{P}(s_1) + \cdots + \mathbf{P}(s_k)$

  - Axiom 3 needs strengthening
  - Do weird sets have probabilities?

Example



- Let every possible outcome have probability 1/16
- $\mathbf{P}(X = 1) =$
- Let $Z = \min(X, Y)$
- $\mathbf{P}(Z = 1) =$
- $\mathbf{P}(Z = 2) =$
- $\mathbf{P}(Z = 3) =$
- $\mathbf{P}(Z = 4) =$

Discrete uniform law

- Let all sample points be equally likely
- Then,

$$\mathbf{P}(A) = \frac{\text{number of elements of } A}{\text{total number of sample points}}$$

- Just count...

A word about infinite sample spaces

- Sample space: $\{1, 2, \ldots\}$
  - We are given $\mathbf{P}(n) = 2^{-n}$, $n = 1, 2, \ldots$
  - Find $\mathbf{P}(\text{outcome is even})$
- Solution:

$$\begin{aligned}\mathbf{P}(\{2, 4, 6, \ldots\}) &= \mathbf{P}(2) + \mathbf{P}(4) + \cdots \\ &= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots = \frac{1}{3}\end{aligned}$$

- Axiom needed:
  If $A_1, A_2, \ldots$ are disjoint events, then:

$$\mathbf{P}(A_1 \cup A_2 \cup \cdots) = \mathbf{P}(A_1) + \mathbf{P}(A_2) + \cdots$$