**Recitation 21**
**May 3, 2005**

1. Define $X$ as the height in meters of a randomly selected Canadian, where the selection probability is equal for each Canadian, and denote $\mathbf{E}[X]$ by $h$. Bo is interested in estimating $h$. Because he is sure that no Canadian is taller than 3 meters, Bo decides to use 1.5 meters as a conservative (large) value for the standard deviation of $X$. To estimate $h$, Bo averages the heights of $n$ Canadians that he selects at random; he denotes this quantity by $H$.

   (a) In terms of $h$ and Bo's 1.5 meter bound for the standard deviation of $X$, determine the expectation and standard deviation for $H$.

   (b) Help Bo by calculating a minimum value of $n$ (with $n > 0$) such that the standard deviation of Bo's estimator, $H$, will be less than 0.01 meters.

   (c) Say Bo would like to be 99% sure that his estimate is within 5 centimeters of the true average height of Canadians. Using the Chebyshev inequality, calculate the minimum value of $n$ that will make Bo happy.

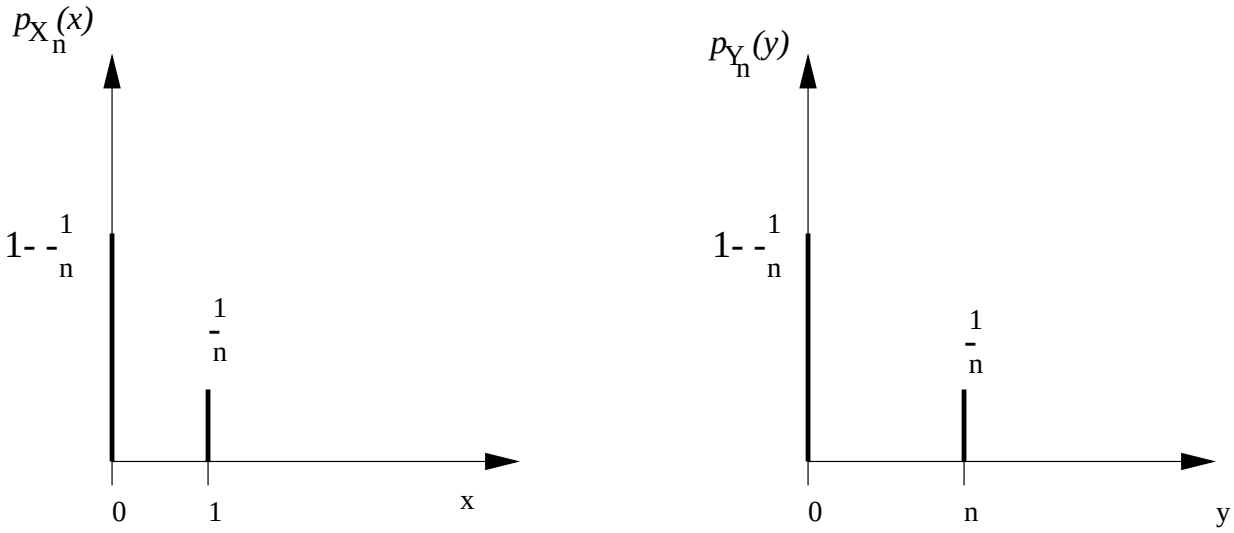
   (d) If we agree that no Canadians are taller than three meters, why is it correct to use 1.5 meters as an upper bound on the standard deviation for $X$, the height of any Canadian selected at random?

2. A storm warning model is 95% accurate. We want to find the probability that out of 50 predictions, at least 45 will be correct.

   (a) Find the above probability by using the normal approximation to the binomial.

   (b) Repeat part (a) this time using the Poisson approximation to the binomial and briefly discuss which one of the answers above you feel to be more accurate, and why.

3.



Let $X_n$ and $Y_n$ have the distributions shown above.

(a) Evaluate the expectation and variance for $X_n$ and $Y_n$.

(b) What does the Chebyshev inequality tell us about the convergence of $X_n$?

(c) What does the Chebyshev inequality tell us about the convergence of $Y_n$?

(d) Is $X_n$ convergent in probability? If so, to what value? Explain.

(e) Is $Y_n$ convergent in probability? If so, to what value? Explain.