Recitation 17
April 14, 2005
More on the Poisson Process

1. Suppose you've bought a new bike at time $t = 0$, where t is our continuous-time variable. Assume the time to the first flat on the front tire, T_F, is exponential(λ_F), the time to first flat on the rear tire, T_R, is exponential(λ_R), and these two random variables are independent of each other.

   (a) Let T be the time to the first flat tire. Find the CDF and PDF of T.

   (b) If tires are repaired in negligible time and are as good as new after that, what is the PMF of the number of times the front tire goes flat by time t? And what is the PMF in the case of the rear tire?

   (c) Find the probability that the front tire is flat, given that you have your first flat tire.

   (d) Write the joint PDF of T_F and T_R, and use it to compute the probability that the front tire goes flat before the rear tire. How does this compare with your answer in (c)?

   (e) Find the conditional PDF of T, given that the front tire goes flat before the rear tire.

   (f) At time $t = 0$, you decide to pick one of the tires at random (with equal probability) and stick phosphorescent stickers along the rim (the stickers do not affect the lifetime of the tire.) What is the PDF and the transform of the time until the tire with stickers on goes flat?

2. Shem, a local policeman, drives from intersection to intersection in times that are iid and all exponentially distributed with parameter λ. At each intersection he observes (and reports) a car accident with probability p. (This activity does not slow his driving at all.) Independently of all else, Shem receives extremely brief radio calls in a Poisson manner with an average rate of μ calls per hour.

   (a) Determine the PMF for N, the number of intersections Shem visits up to and including the one where he reports his first accident.

   (b) Determine the PDF for Q, the length of time Shem drives between reporting accidents.

   (c) What is the PMF for M, the number of accidents which Shem reports in two hours?

   (d) What is the PMF for K, the number of accidents Shem reports between his receipt of two successive radio calls?

   (e) We observe Shem at a random instant long after his shift has begun. Let W be the total time from Shem's last radio call until his next radio call.

      i. What is the PDF of W?

      ii. What is the transform of the PDF of W?

   (f) Again we observe Shem at a random instant. Determine the transform of the PDF for V, the time from our observation until he receives his first radio call *after* his next accident report?