

Recitation 8
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1. A point (X, Y) is chosen from within the circle of radius r and area πr^2 according to the uniform PDF

$$f_{X,Y}(x, y) = \begin{cases} c & , \quad x^2 + y^2 \leq r^2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

where c is an appropriate constant. Recall that the equation of a circle is $x^2 + y^2 = r^2$.

- (a) Are the random variables X and Y independent?
 - (b) Determine the value of the constant c .
 - (c) Find $f_X(x)$.
 - (d) Find $f_{X|Y}(x | y)$ and $E[X | Y = \sqrt{3}r/2]$.
 - (e) Find $f_{X,Y|A}(x, y | A)$ where A is the event $\sqrt{X^2 + Y^2} \leq r/2$.
2. Suppose X_1, \dots, X_n are iid uniform random variables from 0 to 1.
- a) Let $Y = \max\{X_i\}$. Find the CDF of random variable Y .
 - b) Let $W = \min\{X_i\}$. Find the CDF of random variable W .
 - c) Find the joint CDF of random variables Y, W .
3. Imagine a TV game show where each contestant i spins an infinitely calibrated "fair" wheel of fortune, which assigns him/her some real number X_i (not necessarily integer) between 1 and 100.
- a) Find $P(X_1 < X_2)$. Explain your answer.
 - b) Find $P(X_1 < X_2, X_1 < X_3)$, i.e., find the probability that the first contestant will have the smallest value of the first three contestants.
 - c) Consider a new random variable N , which is integer valued. N is the index of the first contestant who is assigned a smaller number than contestant 1. As an illustration, if contestant 1 has a smaller value than contestants 2, 3, and 4, but contestant 5 has a smaller value than contestant 1 ($X_5 < X_1$), then $N = 5$. Find $P(N > n)$ as a function of n .