Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2005)

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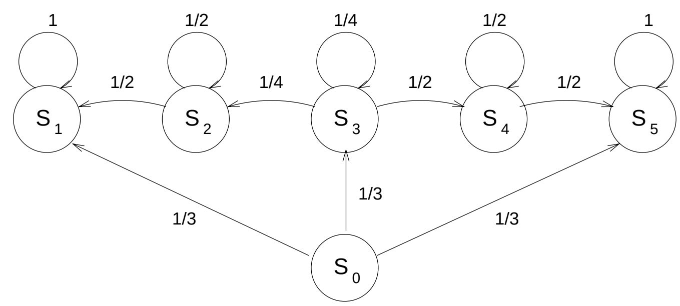
Markov Processes (Section 6.1-6.2)

1. Consider the following Markov chain:



   Given that the above process is in state S_0 just before the first trial, determine by inspection the probability that:

   (a) The process enters S_2 for the first time as the result of the kth trial.
   (b) The process never enters S_4.
   (c) The process enters S_2 and then leaves S_2 on the next trial.
   (d) The process enters S_1 for the first time on the third trial.
   (e) The process is in state S_3 immediately after the nth trial.

2. Eight balls, four red and four blue, are distributed in two urns in such a way that each contains four balls. At each step, we draw one ball from each urn at random and place the ball drawn from the first urn into the second, and conversely with the ball from the second urn.

   (a) Define a set of states that distinguishes among various color combinations of balls in the two urns. What is the smallest number of states necessary?
   (b) Using your set of states, draw an appropriate state transition diagram.
   (c) How many states would you need to distinguish among various color combinations if in addition to the red and blue balls, there are four white balls and the resultant twelve balls are distributed so that there are six balls in each urn?

3. A mouse makes a (Markov) random walk in a tiled corridor, consisting of n tiles $T_1, \ldots, T_n$. (Here, n is an even positive integer.) If the mouse is currently on tile T_i, at the next point in time he will find himself on either tile T_{i+1} or T_{i-1}, with equal probability. The only exceptions are that if he is currently on tile T_1, at the next point in time he will be on tile T_2. Similarly, if he is currently on tile T_n, at the next point in time he will be on tile T_{n-1}.

   We have a monitoring device which at each instant of time k records "L" if the mouse is in the left half of the corridor (tiles $T_1, \ldots, T_{n/2}$) or "R" if the mouse is in the right half of the corridor (tiles $T_{(n/2)+1}, \ldots, T_n$)

   Can the sequence of letters recorded by the monitoring device be modeled as the state history of a Markov process? If yes, derive the transition probabilities. If not, explain why not.