Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2005)

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1. Problem 2.38, page 132 in the text. Alice passes through four traffic lights on her way to work, and each light is equally likely to be green or red, independently of the others.

   (a) What is the PMF, the mean, and the variance of the number of red lights that Alice encounters?

   (b) Suppose that each red light delays Alice by exactly two minutes. What is the variance of Alice's commuting time?

2. Problem 2.40, page 132 in the text. A particular professor is known for his arbitrary grading policies. Each paper receives a grade from the set $\{A, A-, B+, B, B-, C+\}$, with equal probability, independently of other papers. How many papers do you expect to hand in before you receive each possible grade at least once?

3. Problem 2.42, page 133 in the text (partially and slightly modified). **Computational problem.** Here is a probabilistic method for computing the area of a given subset S of the unit square. The method uses a sequence of independent random selections of points in the unit square $[0, 1] \times [0, 1]$, according to a uniform probability law. If the ith point belongs to the subset S the value of a random variable X_i is set to 1, and otherwise it is set to 0. Let $X_1, X_2, \ldots$ be the sequence of random variables thus defined, and for any n, let

$$S_n = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

   (a) Show that $\mathbf{E}[S_n]$ is equal to the area of the subset S, and that $\text{var}(S_n)$ diminishes to 0 as n increases.

   (b) Show that to calculate S_n, it is sufficient to know S_{n-1} and X_n, so that past values of $X_k, k = 1, \ldots, n-1$, do not need to be remembered. Give a formula.

   (c) Suppose you write a computer program to generate S_n for $n = 1, 2, ..., 10000$, using the computer's random number generator, for the case where the subset S is the circle inscribed within the unit square. How can you use your program to measure experimentally the value of π?