Recitation 13
March 31, 2005

1. Give an example of two random variables that are uncorrelated but not independent.

2. Consider n independent tosses of a die. Each toss has probability p_i of resulting in i. Let X_i be the number of tosses that result in i. Show that X_1 and X_2 are negatively correlated (i.e., a large number of ones suggests a smaller number of twos).

3. The value of a random variable X is transmitted, but what we receive (denoted by Y) is the value of X corrupted by some additive noise W; that is, $Y = X + W$. We know the distribution of X and W, and let us assume that both random variables have the same PDF. Suppose also that X and W are independent.

   (a) Let y be the observed value of Y. Produce the least mean-squares estimate of X given Y, i.e., the estimate $\hat{X}$ of X based on y, such that $E[(\hat{X} - X)^2 | Y = y]$ is minimized.

   (b) Suppose X and W had the same distribution, but that they were dependent. What would our new answer be?

4. Problem 4.24, on page 264 in the book. **Schwarz inequality.** Show that if X and Y are random variables, we have
$$(\mathbf{E}[XY])^2 \leq \mathbf{E}[X^2]\mathbf{E}[Y^2].$$