

Problem Set 6
Due: March 30, 2005

1. Find the PDF of $X^2 + 2X + 1$ in terms of the PDF of X . Specialize the answer to the case where X is uniformly distributed between 0 and 1.
2. Let X be uniform on $[0, 2]$ and let Y be uniform on $[3, 4]$. Assume that X and Y independent. Find and sketch the PDF of $X + Y$, using convolutions.
3. Alice and Bob flip bias coins independently. Alice's coin comes up heads with probability $1/4$, while Bob's coin comes up head with probability $3/4$. Each stop as soon as they get a head; that is, Alice stops when she gets a head while Bob stops when he gets a head. What is the PMF of the total amount of flips until both stop? (That is, what is the PMF of the combined total amount of flips for both Alice and Bob until they stop?)

4. Suppose that

$$M_X(s) = \frac{3}{(1-s)(3-s)}.$$

Find a formula for the PDF of X .

5. Let X and Y be independent random variables which have only non-negative integer experimental values. Each part of this question introduces a new random variable. For each part, determine the expectation and variance of the new random variable in terms of $\mathbf{E}[X]$, $\mathbf{E}[Y]$, $\text{var}(X)$, and $\text{var}(Y)$.

Notation: X_1, X_2, \dots are independent random variables with the same distribution as random variable X . Provide lucid explanations for your answers.

- (a) $W = X_1 + X_2 + X_3 + X_4$
 - (b) $V = 0.25(X_1 + X_2 + X_3 + X_4)$
 - (c) $U = X_1 + X_2 + X_3 + X_4 + Y$
 - (d) $R = 4X - Y$
 - (e) $M_Q(s) = [M_X(s)]^5$
 - (f) $M_H(s) = [M_X(s)]^2[M_Y(s)]^3$
 - (g) $M_G(s) = e^{6s}M_X(s)$
 - (h) $M_D(s) = M_X(7s)$
6. Let X be the number of hours Mimi spends in the library. The transform associated with X is given by

$$M_X(s) = \frac{1}{8}e^s + \frac{1}{2}e^{2s} + \frac{5}{24}e^{4s} + \frac{1}{6}e^{6s}$$

Given that the number of hours Mimi stays in the library is x , Mimi will borrow Y books, where Y has a Poisson distribution with parameter x .

- (a) Find the pmf of X .
- (b) Find the transform of the number of books that Mimi borrows.

- (c) Find the expectation and variance of the number of books that Mimi borrows.
7. Let X , Y and Z be independent zero-mean Gaussian random variables with variance σ_X^2 , σ_Y^2 and σ_Z^2 respectively. Let $Q = aX + bY + cZ$, where a , b and c are real constants. Find the PDF of Q .
8. Let X be a continuous random variable that is uniformly distributed on the interval $[a, b]$. Find $\mathbf{E}[X]$ with and without using transforms.