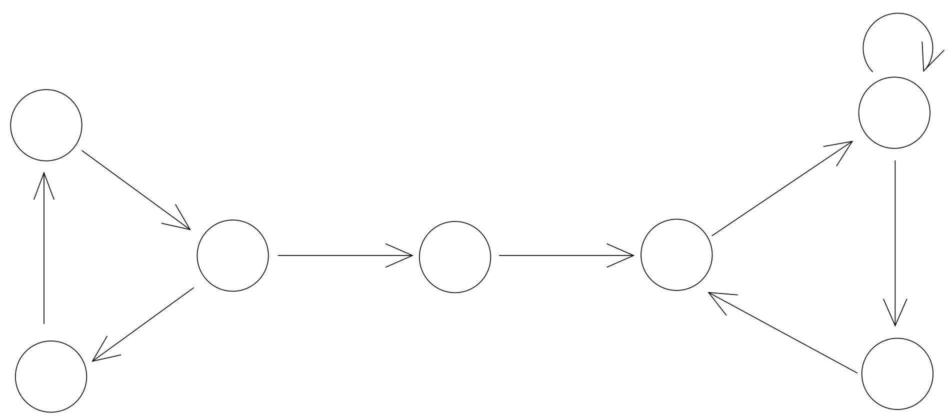
Recitation 19
April 26, 2005
Markov Processes (Section 6.3)

1. For a Markov chain with a finite number of states, we define

$$r_{ij}(n) \triangleq \mathbf{P}(X_n = j | X_0 = i).$$

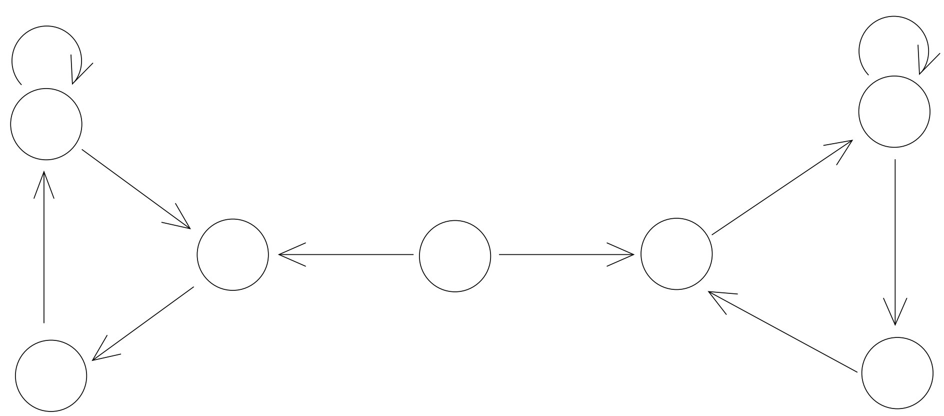
We define π_j as the steady state probability of being in state j, **provided $r_{ij}(n)$ converges as $n \to \infty$ and the value to which it converges does not depend on the starting state i.** The arrows correspond to positive single-step transition probabilities. Determine if each statement is true or false for each of the chains below.

(a)



(i) $r_{ij}(n)$ converges to a limit as $n \to \infty$ for each pair of states (i, j).
(ii) Statement (i) is true and $\lim_{n\to\infty} r_{ij}(n) = \pi_j$ (not varying with i) for each pair of states (i, j).
(iii) Statement (ii) is true and $\pi_j > 0$ for each state j.

(b)



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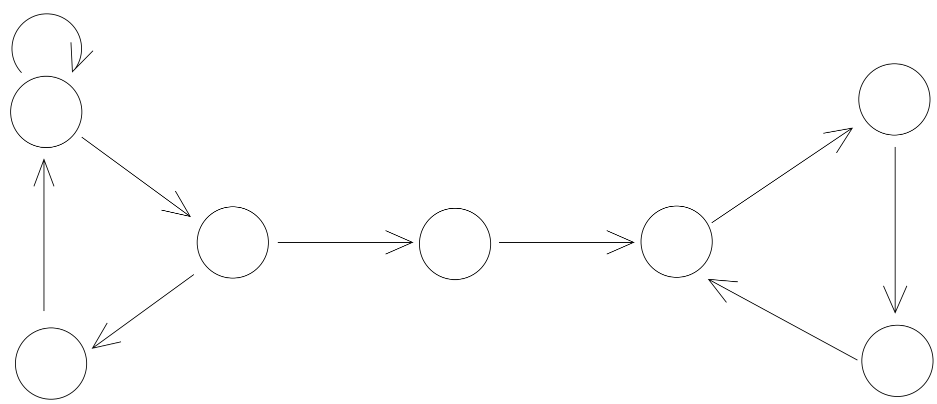

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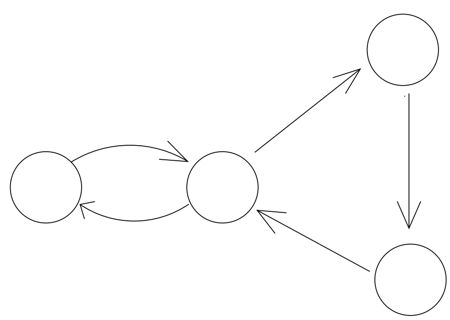

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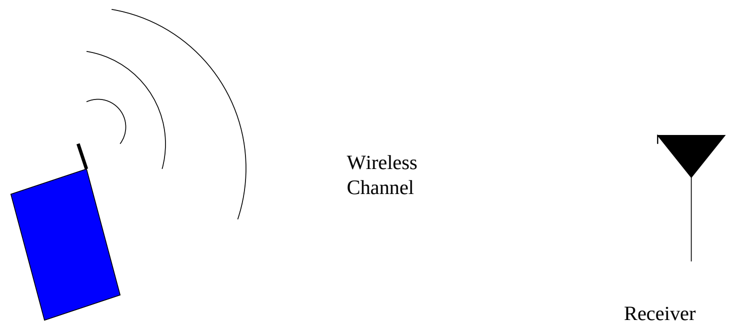
2. A PDA/phone transmits one packet in every time slot over a wireless connection. Every packet, independently of any other packet, is received in error with probability p. To avoid wasting transmitter power when the link quality is poor, the transmitter enters a "timeout" state whenever five consecutive packets are received in error. During such a timeout, the mobile terminal performs an independent Bernoulli trial with success probability q in every slot. When a success occurs, the mobile terminal starts transmitting in the next slot as though no packets had been in error.



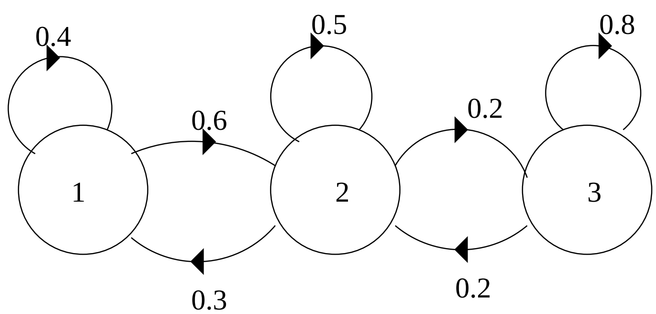
(a) Construct a discrete-time Markov chain for this system, which includes

  i. defining an appropriate state space and

  ii. drawing the transition probability graph.

(b) Solve for the steady-state probabilities in terms of parameters p and q.

3. Consider the Markov chain below. Let us refer to a transition that results in a state with a higher (respectively, lower) index as a birth (respectively, death). Calculate the following quantities, assuming that when we start observing the chain, it is already in steady-state.



(a) For each state i, the probability that the current state is i.

(b) The probability that the first change of state we observe is a birth.

(c) The conditional probability that the process was in state 2 before the first transition that we observe, given that this transition was a birth.

(d) The conditional probability that the first observed transition is a birth given that it resulted in a change of state.

(e) The conditional probability that the first observed transition leads to state 2, given that it resulted in a change of state.