

**Tutorial 1**  
**Week of February 7, 2005**

1. Let  $A$  and  $B$  be events such that  $A \subset B$ . Can  $A$  and  $B$  be independent?
2. (**THE CHESS PROBLEM**) This year's Belmont chess champion is to be selected by the following procedure. Bo and Ci, the leading challengers, first play a two-game match. If one of them wins both games, he gets to play a two-game *second round* with Al, the current champion. Al retains his championship unless a second round is required and the challenger beats Al in both games. If Al wins the initial game of the second round, no more games are played.

Furthermore, we know the following:

- The probability that Bo will beat Ci in any particular game is 0.6.
- The probability that Al will beat Bo in any particular game is 0.5.
- The probability that Al will beat Ci in any particular game is 0.7.

Assume no tie games are possible and all games are independent.

- (a) Determine the apriori probabilities that
    - i. the second round will be required.
    - ii. Bo will win the first round.
    - iii. Al will retain his championship this year.
  - (b) Given that the second round is required, determine the conditional probabilities that
    - i. Bo is the surviving challenger.
    - ii. Al retains his championship.
  - (c) Given that the second round was required and that it comprised only one game, what is the conditional probability that it was Bo who won the first round?
3. Imno Nerd, an MIT Freshman, makes one to five new friends every week, with equal probability. The number of friends she makes during each week is independent from all other weeks. We are concerned with two consecutive weeks.

Let event  $A$  be "Imno made a total of 10 friends during the two weeks". Let event  $B$  be "Imno made more than 5 friends during the two weeks."

- (a) Are events  $A$  and  $B$  independent?
- (b) Let  $C$  be the event "Imno made exactly 5 friends during the first week". Are  $A$  and  $B$  independent, conditioned on  $C$ ?
- (c) Is  $A$  independent of  $C$ ? Is  $B$  independent of  $C$ ?
- (d) Given that Imno made a total of 6 friends in two weeks, what is the probability that she made exactly 2 friends in the first week? How about 3 friends in the first week?
- (e) Optional. Given that Imno made a total of 6 friends in two weeks, what is the probability that she made exactly 2 friends in at least one of the weeks? How about exactly 3 friends in at least one of the weeks?