Recitation 10
March 15, 2005

1. Consider two independent and identically distributed discrete random variables X and Y. Assume that their common PMF, denoted by $p_X(x)$, is symmetric around zero, i.e., $p_X(x) = p_X(-x)$ for all x. Show that the PMF of $X + Y$ is also symmetric around zero and is largest at zero. *Hint:* Use the Schwarz inequality: $\sum_k (a_k b_k) \leq \left(\sum_k a_k^2\right)^{1/2} \left(\sum_k b_k^2\right)^{1/2}$.

2. Adapted from Problem 2.40, page 132 in the text. A particular professor is known for his arbitrary grading policies. Each paper receives a grade from the set $\{A, B, C\}$, with equal probability, independently of other papers. What is the PMF of the number of papers that you hand in before you receive each possible grade at least once?

3. Alice and Bob both toss 3 coins n times, with $n \geq 5$. Each coin has a probability p of coming up heads. At each time, they win \$1 if they toss at least 2 heads. Let X and Y be the amount of money Alice and Bob have after n tosses.

   (a) Find the PMF $p_X(x)$ and $p_Y(y)$.

   (b) Find the PMF of the total amount of money Alice and Bob make.

   (c) Given that they make at least \$10 combined, find the PMF of the total amount of money Alice and Bob make.