Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2005)

Recitation 2
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1. Example 1.18, page 33 in the text. A test for a certain rare disease is assumed to be correct 95% of the time: if a person has the disease, the test results are positive with probability 0.95, and if the person does not have the disease, the test results are negative with probability 0.95. A random person drawn from a certain population has probability 0.001 of having the disease. Given that the person just tested positive, what is the probability of having the disease?

2. **Communication through a noisy channel.** Problem 1.27, page 59 in the text. A binary (0 or 1) symbol transmitted through a noisy communication channel is received incorrectly with probability e_0 and e_1, respectively, as described in Figure 1. Errors in different symbol transmissions are independent.

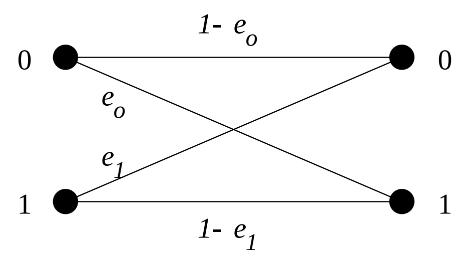

Figure 1: Description of a binary channel

   (a) Suppose that the channel source transmits a 0 with probability p and transmits a 1 with probability $1 - p$. What is the probability that a randomly chosen symbol is received correctly?

   (b) Suppose that the string of symbols 1011 is transmitted. What is the probability that all the symbols in the string are received correctly?

   (c) In an effort to improve reliability, each symbol is transmitted three times and the received symbol is decoded by a majority rule. In other words, a 0 (or 1) is transmitted as a 000 (or 111, respectively), and it is decoded at the receiver as a 0 (or 1) if and only if the received three-symbol string contains at least two 0s (or 1s, respectively). What is the probability that a transmitted 0 is correctly decoded?

   (d) Suppose that the source transmits a 0 with probability p and transmits a 1 with probability $1 - p$, and the scheme of part (c) is used. What is the probability that a 0 was transmitted given that the received string is 101?

3. Problem 1.30, page 60 in the text. An electrical system consists of identical components that are operational with probability p, independently of other components. The components are connected in three subsystems, as shown in Figure 2. The system is operational if there is a path that starts at point A, ends at point B, and consists of operational components. This is the same as requiring that all three subsystems are operational. What are the probabilities that the three subsystems, as well as the entire system, are operational?

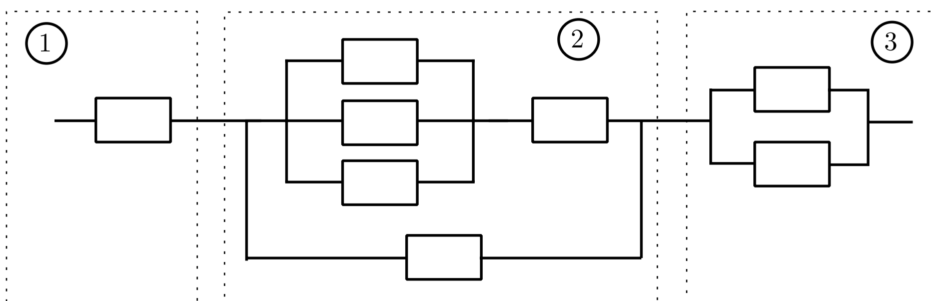

Figure 2: Diagram of an electrical system