

**Poisson process review and examples**

- **Readings:** Finish Section 5.2.
- Review of Poisson process
- Examples
- Random incidence

- Numbers of arrivals in disjoint time intervals are independent

$$P(k, \delta) \approx \begin{cases} 1 - \lambda\delta & \text{if } k = 0 \\ \lambda\delta & \text{if } k = 1 \\ 0 & \text{if } k > 1 \end{cases}$$

$$P(k, \tau) = \frac{(\lambda\tau)^k e^{-\lambda\tau}}{k!}, \quad k = 0, 1, \dots$$

$$E[N] = \lambda\tau$$

- Interarrival times ( $k = 1$ ): exponential

$$f_{T_1}(t) = \lambda e^{-\lambda t}, \quad t \geq 0, \quad E[T_1] = 1/\lambda$$

- Time  $Y_k$  to  $k$ th arrival:

$$f_{Y_k}(y) = \frac{\lambda^k y^{k-1} e^{-\lambda y}}{(k-1)!}, \quad y \geq 0$$

**Poisson catches**

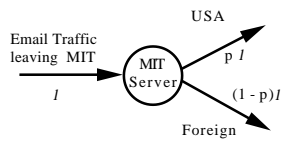
- Assume: Poisson,  $\lambda = 0.6/\text{hour}$   
Fish for two hours; if no catch, continue until first catch.
- A.**  $P(\text{fish for more than two hours}) =$
  - B.**  $P(\text{fish for more than two and less than five hours}) =$
  - C.**  $P(\text{catch at least two fish}) =$
  - D.**  $E[\text{number of fish}] =$
  - E.**  $E[\text{future fishing time} \mid \text{fished for four hours}] =$
  - F.**  $E[\text{total fishing time}] =$

**Light bulb example**

- Each light bulb has independent, exponentially distributed lifetime  $\lambda e^{-\lambda t}$ ,  $t \geq 0$
- Install three light bulbs.  
Find expected time until last light bulb dies out

## Splitting of Poisson processes

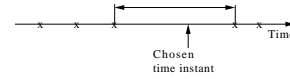
- Each message is routed along the first stream with probability  $p$ 
  - Routings of different messages are independent



- Each output stream is Poisson

## Random incidence for Poisson

- Poisson process that has been running forever
- Show up at some "random time" ("random incidence")



- What is the distribution of the length of the chosen interarrival interval?

## Renewal processes

- Series of successive arrivals
  - i.i.d. interarrival times (but not necessarily exponential)
- **Example:**
  - Bus interarrival times are equally likely to be 5 or 10 minutes
- If you arrive at a "random time":
  - what is the probability that you selected a 5 minute interarrival interval?
  - what is the expected time to next arrival?