Tutorial 4
Week of February 28, 2005

1. In a particular road race, the runner-up is given a reward conditional on how close to the winner he was. He is given 10 dollars for being within one minute of the winner, 6 dollars for being within one to three minutes of the winner, 2 dollars for being within 3 to 6 minutes of the winner, and nothing otherwise. Given that the time the runner-up finishes after the winner is uniformly distributed between 1 and 12, find the runner-up's expected reward.

2. Random variables X and Y are distributed according to the joint PDF

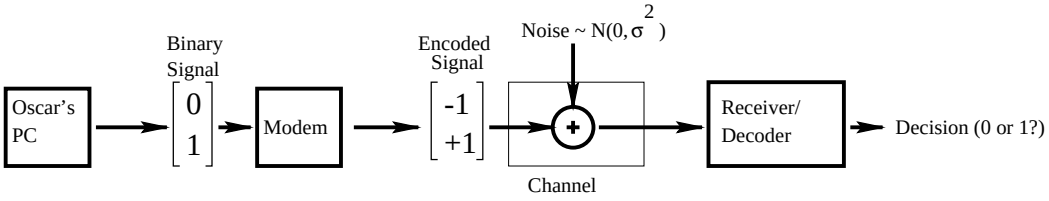
$$f_{X,Y}(x,y) = \begin{cases} ax, & \text{if } 1 \le x \le y \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Evaluate the constant a.

   (b) Determine the marginal PDF $f_Y(y)$.

   (c) Determine the expected value of $\frac{1}{X}$, given that $Y = \frac{3}{2}$.

3. Wanting to browse the net, Oscar uses his high-speed 300-baud modem to connect through his Internet Service Provider. The modem transmits bits in such a fashion that -1 is sent if a given bit is zero and +1 is sent if a given bit is one. The telephone line has additive zero-mean Gaussian (normal) noise with variance σ^2 (so, the receiver on the other end gets a signal which is the sum of the transmitted signal and the channel noise). The value of the noise is assumed to be independent of the encoded signal value.



   We assume that the probability of the modem sending -1 is p and the probability of sending 1 is $1 - p$.

   (a) Suppose we conclude that an encoded signal of -1 was sent when the value received on the other end of the line is less than a (where $-1 < a < +1$), and conclude $+1$ was sent when the value is more than a. What is the probability of making an error?

   (b) Answer part (a) assuming that $p = 2/5$, $a = 1/2$ and $\sigma^2 = 1/4$.