

Recitation 9
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1. Suppose X is uniformly distributed between 0 and 1. Find the density function for random variable Y where:

(a) $Y = X^2$.

(b) $Y = e^X$.

2. Suppose the ordered pair (X, Y) has joint density function:

$$f_{X,Y}(x, y) = \frac{1}{\pi}, \text{ for } x^2 + y^2 \leq 1$$

i.e. (X, Y) are the coordinates of a point uniformly distributed on the unit circle centered at $(0, 0)$. If we have:

$$\begin{aligned} R &= \sqrt{X^2 + Y^2} \\ \Theta &= \tan^{-1} \frac{Y}{X} \end{aligned}$$

find the joint density function: $f_{R,\Theta}(r, \theta)$.

3. Random variables X and Y have the joint PDF:

$$f_{X,Y}(x, y) = \begin{cases} 2 & \text{if } x > 0 \text{ and } y > 0 \text{ and } x + y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- Let A be the event $Y \leq 0.5$.
 - Let B be the event $Y > X$.
 - Please note (happily!) that because the density function is uniform over the indicated region, probabilities of events defined on this region are proportional to areas.
 - Please simplify your expressions for your answers as much as possible.
- (a) Determine the numerical value of $\mathbf{P}(B|A)$.
- (b) Prepare a neat, fully labeled sketch of $f_{X|Y}(x|0.5)$. Also evaluate the conditional expectation and the conditional variance for X , given that the experimental value of Y is equal to 0.5.
- (c) Let $T = XY$. Determine the numerical value of $\mathbf{E}[T]$.