Tutorial 9
Week of April 11, 2005

1. Transmitters A and B independently send messages to a single receiver in a Poisson manner with average message arrival rates of λ_A and λ_B, respectively. All messages are so brief that we may safely assume that they occupy only single points in time.

   The number of words in every message, regardless of its transmitting source, may be considered to be an independent experimental value of random variable W with PMF

$$p_W(w) = \begin{cases} 2/6 & , \quad w = 1 \\ 3/6 & , \quad w = 2 \\ 1/6 & , \quad w = 3 \\ 0 & , \quad \text{otherwise} \end{cases}$$

   (a) What is the probability that, during an interval of duration t, a total of exactly nine messages will be received?

   (b) Let N be the total number of words received during an interval of duration t. Determine the expected value for random variable N.

   (c) Determine the PDF for X, the time from $t = 0$ until the receiver has received exactly eight three-word messages from transmitter A.

   (d) Independent of what happens to all other words, a transmitter damages any particular word it sends with probability 10^{-3}. What is the probability that any particular damaged word is part of a three-word message?

   (e) What is the probability that exactly eight of the next twelve messages received will be from transmitter A?

2. The MIT soccer team needs at least 8 players to avoid forfeiting a game. Assume that each player has some chance of being eliminated for the remainder of the season owing to injury, and that her playing lifetime for a given season is exponentially distributed with parameter λ. For simplicity, assume that the coach insists on only playing 8 players at a time, and then replaces a player as soon as she gets hurt. Find

   (a) The expected time until the first substitution.

   (b) The distribution of total time the team can play in a season, given that there are n women on the team.