

Introduction to Probability

1 Probability

Probability will be the topic for the rest of the term. Probability is one of the most important subjects in Mathematics and Computer Science. Most upper level Computer Science courses require probability in some form, especially in analysis of algorithms and data structures, but also in information theory, cryptography, control and systems theory, network design, artificial intelligence, and game theory. Probability also plays a key role in fields such as Physics, Biology, Economics and Medicine.

There is a close relationship between Counting/Combinatorics and Probability. In many cases, the probability of an event is simply the fraction of possible outcomes that make up the event. So many of the rules we developed for finding the cardinality of finite sets carry over to Probability Theory. For example, we'll apply an Inclusion-Exclusion principle for *probabilities* in some examples below.

In principle, probability boils down to a few simple rules, but it remains a tricky subject because these rules often lead unintuitive conclusions. Using "common sense" reasoning about probabilistic questions is notoriously unreliable, as we'll illustrate with many real-life examples.

This reading is longer than usual . To keep things in bounds, several sections with illustrative examples that do not introduce new concepts are marked "[Optional]." You should read these sections selectively, choosing those where you're unsure about some idea and think another example would be helpful.

2 Modelling Experimental Events

One intuition about probability is that we want to predict how likely it is for a given experiment to have a certain kind of outcome. Asking this question invariably involves four distinct steps:

Find the sample space. Determine all the possible outcomes of the experiment.

Define the event of interest. Determine which of those possible outcomes is "interesting."

Determine the individual outcome probabilities. Decide how likely each individual outcome is to occur.

Determine the probability of the event. Combine the probabilities of "interesting" outcomes to find the overall probability of the event we care about.

In order to understand these four steps, we will begin with a toy problem. We consider rolling three dice, and try to determine the probability that we roll exactly two sixes.