Massachusetts Institute of Technology
6.042J/18.062J, Fall '02: Mathematics for Computer Science
Professor Albert Meyer and Dr. Radhika Nagpal

In-Class Problems — Week 9, Fri

Problem 1. Recall the definition of a multinomial coefficient:

$$\binom{n}{r_1, r_2, \ldots, r_k} ::= \frac{n!}{r_1!\, r_2! \cdots r_k!}. \tag{1}$$

An alternative definition is

$$\binom{n}{r_1, r_2, \ldots, r_k} ::= \binom{n}{r_1} \cdot \binom{n-r_1}{r_2} \binom{n-r_1-r_2}{r_3} \cdots \binom{n-r_1-r_2-\cdots-r_{k-1}}{r_k} \tag{2}$$

Prove that the two definitions are equivalent using both a (i) combinatorial and (ii) an algebraic argument.

Problem 2. Suppose that we have a generalized n-letter "MISSISSIPPI" word whose letters break down as follows: m occurrences of "M", i occurrences of "I", s of "S", and p of "P". For example, for the word "MISSISSIPPI" breaks down as $(n; m, i, s, p) = (11, 1, 4, 4, 2)$. For each of the following formulas, identify which correctly describe the number of different permutations of the generalized "MISSISSIPPI" word. For the correct formulas, justify your answer using a combinatorial (not an algebraic) argument.

(a) $\binom{n}{m, i, s, p}$.

(b) $\binom{n}{p, i, m, s}$

(c) $\binom{n}{m, i, s, n-m-i-s}$

(d) $\binom{n}{m}\binom{n}{i}\binom{n}{s}\binom{n}{p}$

(e) $\binom{n}{m}\binom{m}{i}\binom{i}{s}\binom{s}{p}$

Problem 3. The U.S. House of Representatives has 435 seats. (In this context, a "seat" is not a chair, but a right to belong to the legislative body. Thus, all seats are indistinguishable.)

(a) In how many ways can the seats be distributed among three political parties?

(b) In how many ways can the seats be distributed among three political parties so that no party has a majority? (This implies that a coalition of any two parties *does* form a majority.)

(c) How do your solutions to parts (a) and parts (b) change if the "seats" are actually distinguishable chairs (numbered 1 through 435)?