

Sums, Products & Asymptotics

1 Closed Forms and Approximations

Sums and products arise regularly in the analysis of algorithms and in other technical areas such as finance and probabilistic systems. We've already seen that

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

Having a simple *closed form* expression such as $n(n+1)/2$ makes the sum a lot easier to understand and evaluate. We proved by induction that this formula is correct, but not where it came from. In Section 4, we'll discuss ways to find such closed forms. Even when there are no closed forms exactly equal to a sum, we may still be able to find a closed form that *approximates* a sum with useful accuracy.

The product we focus on in these notes is the familiar factorial:

$$n! ::= 1 \cdot 2 \cdots (n-1) \cdot n = \prod_{i=1}^{n} i.$$

We'll describe a closed form approximation for it called *Stirling's Formula*.

Finally, when there isn't a good closed form approximation for some expression, there may still be a closed form that characterizes its growth rate. We'll introduce *asymptotic notation*, such as "big Oh", to describe growth rates.

2 Annuities

Would you prefer a million dollars today or $50,000 a year for the rest of your life? On the one hand, instant gratification is nice. On the other hand, the total dollars received at $50K per year is much larger if you live long enough.

Formally, this is a question about the value of an annuity. An *annuity* is a financial instrument that pays out a fixed amount of money at the beginning of every year for some specified number of years. In particular, an n-year, m-payment annuity pays m dollars at the start of each year for n years. In some cases, n is finite, but not always. Examples include lottery payouts, student loans, and home mortgages. There are even Wall Street people who specialize in trading annuities.