Massachusetts Institute of Technology
6.042J/18.062J, Fall '02: Mathematics for Computer Science
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In-Class Problems — Week 8, Mon

Problem 1. Let S be the set of all 4-digit base-10 numbers that contain the digit 7 somewhere. (Numbers can start with a 0, and thus $0007 \in S$.) The size of S can be determined in several different ways.

(a) Determine $|S|$ by counting the number of all possible 4-digit numbers minus the number of those that do *not* contain 7.

(b) Determine $|S|$ by counting the set of numbers where the first 7 occurs in the first digit, where the first 7 appears as the second digit, where the first 7 appears as the third digit, and where the first 7 appears as the last digit.

(c) Determine $|S|$ by counting the number of 4-digit numbers with exactly one 7, with exactly two 7's, with exactly three 7's and with exactly four 7's.

(d) Argue in each case that the counting method is correct by arguing that (1) everything has been counted at least once (2) nothing has been counted twice.

Problem 2. Each of the 49 students in an MIT class understands mathematical equations, exercises regularly, or loves literature. Of these,

- 20 students understand mathematical equations,
- 20 students exercise regularly,
- 26 students love literature,
- 36 students understand mathematical equations or exercise regularly,
- 38 students understand mathematical equations or love literature, and
- 40 students exercise regularly or love literature.

(a) How many students understand mathematical equations *and* exercise regularly *and* love literature?

(b) Suppose you don't know the number of students in the class, but the rest of the information is the same. What is the largest possible number of students who understand mathematical equations *and* exercise regularly *and* love literature? For this situation, how many students are in the class?