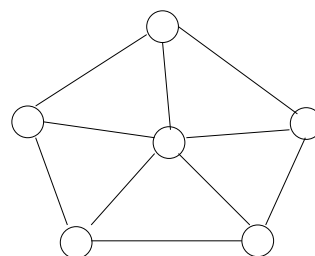
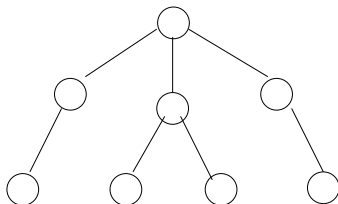
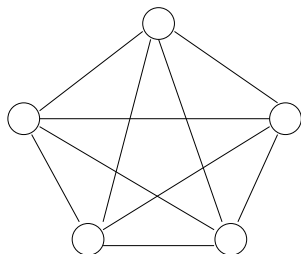
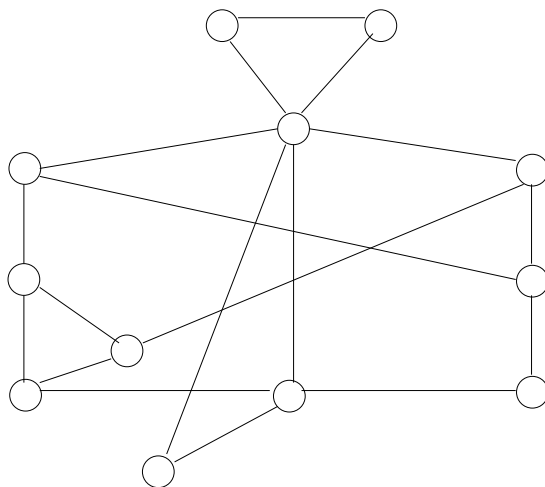
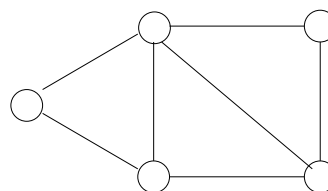
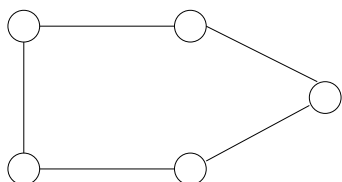


In-Class Problems — Week 4, Wed

Problem 1. Color the following graphs with the minimum possible colors, such no two adjacent vertices have the same color.



Definition: The degree of a vertex is the number of edges connecting it to other vertices.

Theorem: The sum of the degrees of the vertices in a simple graph equals twice the number of edges, i.e. $2|E| = \sum_{v \in V} \deg(v)$.

Handshaking Theorem: In every graph, there are an even number of vertices of odd degree.

Problem 2. Prove that in any undirected graph, the sum of the squares of the vertex degrees is even, or provide a counterexample.

Problem 3. Prove that it is possible to color a graph G with $d_{max} + 1$ colors, where d_{max} represents the maximum degree of any node in G .