Mathematics for Computer Science
MIT 6.042J/18.062J

Introduction to Probability Theory

Probability: First Idea

- set of basic experimental *outcomes,*
- some subset of outcomes considered a noteworthy *event.*
- $\text{Probability}\{\text{event}\} ::= \dfrac{\#\text{outcomes in event}}{\text{total }\#\text{ outcomes}}$

Counting in Probability

What is the *probability* of getting

exactly two jacks

in a poker hand?

Counting in Probability

Outcomes: $\binom{52}{5}$ 5-card hands

Event: $\binom{4}{2} \cdot \binom{52-4}{3}$ hands w/2 Jacks.

$$\text{pr}\{2J\} ::= \frac{\binom{4}{2} \cdot \binom{48}{3}}{\binom{52}{5}} \approx 0.04$$

In-class Problem

Problem 1

The Monty Hall Game

Applied Probability Theory:

Let's Make A Deal

(1970's TV Game Show)

INTRODUCING….

Our own Carol (Karen)

Our own Monty (Josh)

Analyzing Monty Hall

Marilyn Vos Savant tried to explain the Game in her magazine column. She was bombarded by letters (some from Ph.D. mathematicians) disputing her analysis. Debate was between

1) sticking & switching equally good,
2) switching is better.

Analyzing Monty Hall

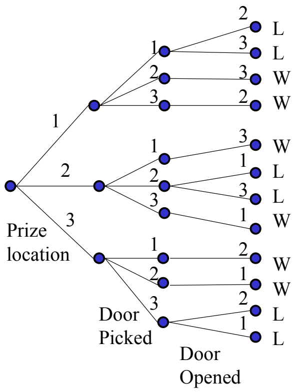
Will illustrate arguments in favor of "equally good":

Analyzing Monty Hall

Determine the *outcomes*:

using a tree showing possible steps often helps

Monty Hall SWITCH Strategy



SWITCH
Wins: 6

Lose: 6

Monty Hall STICK Strategy

Contestant *wins* by *sticking* iff *loses* by *switching*.

STICK
Lose: 6

Wins: 6

Analyzing Monty Hall

CONCLUSION(?!?):
Sticking and Switching are equally good.
Contestant has probability $\frac{1}{2}$ of winning either way.

Analyzing Monty Hall

Intuitive way to see this:
We know Carol will open a goat door, so we learn nothing when she does.

Analyzing Monty Hall

But given that one door has a goat, the other two doors are equally likely to have the prize.
Therefore,
contestant has probability $\frac{1}{2}$ of winning either way (?!?).

Analyzing Monty Hall

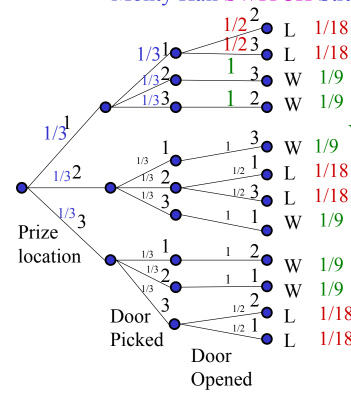
We'll collect data by giving a few more contestants a chance to play.

Analyzing Monty Hall

Wait! All the previous reasoning was wrong!

Look at the outcome tree more carefully:

Monty Hall SWITCH Strategy



W: 6/9 = 2/3

L: 6/18 = 1/3

Probability: Second Idea

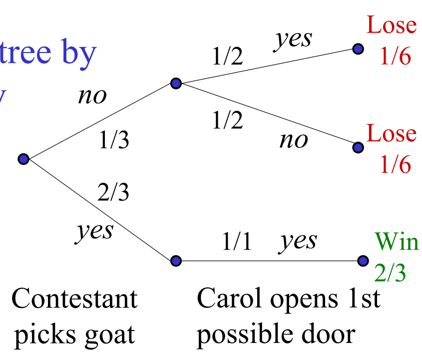
Outcomes may have

differing probabilities!

They need *not* be uniform.

Monty Hall SWITCH Strategy

Simplify tree by symmetry



Monty Hall STICK Strategy

Intuitive reasoning:

1/3 chance of first picking the prize door. This is the *only way* to win with the stick strategy. Otherwise,

switch wins: 2/3 chance

Analyzing Monty Hall

But *intuition*, while important, is *dangerous*.

Stick with the method:

1. Identify outcomes (*tree helps*)
2. Identify event (*e.g. winning*)
3. Assign outcome probabilities
4. Compute event probabilities

Problems

2 & 3