Massachusetts Institute of Technology
6.042J/18.062J, Fall '02: Mathematics for Computer Science
Professor Albert Meyer and Dr. Radhika Nagpal

In-Class Problems — Week 1, Wed

Problem 1. Generalize the proof from lecture[1] that $\sqrt{2}$ is irrational, *e.g.*, how about $\sqrt[3]{2}$?

Problem 2. Albert & Radhi announce that they plan a surprise 6.042 quiz next week. Their students wonder if the quiz could be next Friday. The students realize that it obviously cannot, because if it hadn't been given before Friday, everyone would know that there was only Friday left on which to give it, so it wouldn't be a surprise any more.

So the students ask whether Albert & Radhi could give the surprise quiz Thursday? They observe that if the quiz wasn't given *before* Thursday, it would have to be given *on* the Thursday, since they already know it can't be given on Friday. But having figured that out, it wouldn't be a surprise if the quiz was on Thursday either. Similarly, the students reason that the quiz can't be on Wednesday, Tuesday, or Monday. Namely, it's impossible for Albert & Radhi to give a surprise quiz next week. All the students now relax having concluded that Albert & Radhi must have been bluffing.

And since no one expects the quiz, that's why, when Albert & Radhi give it on Tuesday next week, it really is a surprise!

What do you think is wrong with the students' reasoning? Can you fix it?

[1]See attached. This one not a false proof.

Appendix

Theorem. $\sqrt{2}$ *is an irrational number.*

Remember that an irrational number is a number that can not be expressed as a ratio of two integers.

Proof. The proof is by contradiction. Assume for purpose of contradiction that $\sqrt{2}$ is rational.

Then we can write $\sqrt{2} = m/n$ where m and n are integers and the fraction is in lowest terms. Squaring both sides gives $2 = m^2/n^2$, so $2n^2 = m^2$. This implies that m^2 is even, and hence that m is even; that is, m is a multiple of 2. But that means m^2 is actually a multiple of 4, say $m^2 = 4k$.

Now we have $2n^2 = m^2 = 4k$, so $n^2 = 2k$. So n^2 is even, and hence n is even. But since m and n are both even, the fraction m/n is not in lowest terms, a contradiction. □