

In-Class Problems — Week 1, Fri

Problem 1. Translate the parts of the following deductive arguments into propositional logic notation using logical operators:

$$\begin{aligned}\wedge &::= \text{AND,} \\ \vee &::= \text{OR,} \\ \neg &::= \text{NOT,} \\ \longrightarrow &::= \text{IMPLIES,} \\ \longleftrightarrow &::= \text{IFF (if and only if)}\end{aligned}$$

This may require that you “pin down” a statement that could be interpreted in more than one way. Identify the antecedents and conclusions of the arguments, and determine which are sound deductions and which are not. If the deduction is unsound, demonstrate a possible scenario in which all the antecedents hold but the conclusion does not.

Note: There are several inequivalent, reasonable ways to interpret several of these statements.

(a) The main course will be beef or fish. The vegetable will be peas or corn. We will not have both fish as a main course and corn as a vegetable. Therefore, we will not have both beef as a main course and peas as a vegetable.

(b) Either John or Bill is telling the truth. Either Sam or Bill is lying. Thus, either John is telling the truth or Sam is lying.

(c) Either sales will go up and the boss will be happy, or expenses will go up and the boss won't be happy. Therefore, sales and expenses will not both go up.

Problem 2. Are the following specifications¹ consistent?

1. If the file system is not locked, then
 - (a) new messages will be queued.
 - (b) new messages will be sent to the messages buffer.

(c) the system is functioning normally, and conversely.

2. If new messages are not queued, then they will be sent to the messages buffer.
3. New messages will not be sent to the message buffer.

(a) Begin by translating the parts of the specification into propositional formulas using four propositional variables:

$L ::=$ file system locked,
 $Q ::=$ new messages are queued,
 $B ::=$ new messages are sent to the message buffer,
 $N ::=$ system functioning normally.

(b) The specification is consistent if there is an assignment of truth values to the variables that makes every expression true. Use a truth table to determine whether the specification is consistent.

(c) Use simple reasoning by cases to find a truth assignment that confirms that the system specification of Problem 2. is consistent. Explain why there is only one such assignment.

Problem 3. [Optional] Suppose x is a real number. Prove by cases that there is a real y such that $\frac{y+1}{y-2} = x$ if and only if $x \neq 1$.