Massachusetts Institute of Technology
6.042J/18.062J, Fall '02: Mathematics for Computer Science
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Solutions to In-Class Problems — Week 6, Mon

Problem 1. (Carried over from last Friday (Week 5).)

In Week 5 Notes we considered win-lose games, ignoring games which can end in a draw. Such games have finite-path game trees in which the leaves are labelled with *win*, *lose*, or *draw*, indicating the outcome for the first player.

While a winning stategy for a player ensures the player will win no matter what moves the other player makes, there is now the possibility of a *non-losing* stategy which ensures that the player will *win or draw* no matter what moves the other player makes. (A winning strategy counts as a kind of a non-losing strategy.)

For example, in tic-tac-toe there is no winning strategy for either player, but *both* players have non-losing strategies. For chess, no one knows if white or black has a winning strategy. However, on general principles, we can be sure that at least one player has a non-losing chess strategy.

Explain why for any two-person win-lose-draw game with a finite path game tree, either one player has a winning strategy or *both* players have non-losing strategies.

Solution. We could prove this the same way we proved the Fundamental Theorem for win-lose games, using the well-foundedness of finite-path trees under the "strict-subtree" partial order, but it's more elegant to deduce this as a Corollary of the Fundamental Theorem.

Call the players C and D, and suppose neither one has a winning strategy for the game, T. We want to show that then *both* players have non-losing strategies for T.

The proof is based on the simple observation that in a game with no draws, a non-losing strategy must be a winning strategy.

To prove that C has a non-losing strategy for the game, T, let's consider another game, T_C, identical to T except that draws are considered wins for player C. Since there are no draws in T_C, we conclude that either C or D has a winning strategy for T_C. But a winning strategy for D in the T_C game would also be a winning strategy for D in the original game, T, because wins for D are the same in both games. But we assumed that D has no winning strategy for T, so it doesn't have one for T_C.

That means that C must have the winning strategy for T_C. But a winning strategy for C in T_C would also be a non-losing strategy for C in the original game, T, because wins for C in T_C are always wins for C or draws in T. So we conclude that C has a non-losing strategy for T.

Notice that we haven't assumed that C is the first or second player, so the preceding argument applies equally well to show that D also has a non-losing strategy for T, as required. ■