Problem Set 1

Due: September 9

Reading: Course Overview & Guide, CourseObjectives and Outcomes, Week 1 Notes, Rosen §§1.1–6; §2.3; §2.4 through Lemma 1; §3.1 through Example 26

There is an unusually large amount of reading for this first problem set. We expect that most of the reading in Rosen will be review, and you should quickly skim material you are already familiar with. However, you should also note any important definitions and facts which seem unfamiliar and then *send email* not later than **11am next Monday**

briefly indicating the names *and page numbers* of the unfamiliar material.

There are also a larger than usual number of problems on this problem set. They are intended to be easy and short while highlighting the key material in the reading. We don't want you to spend more than five (5) hours on this first assignment, including the reading. Try to do most of the problems, but don't worry if you run out of time to complete them all; this first problem set will be graded very generously.

Problem 1. Identify exactly where the bugs are in each of the following false proofs.

(a) [1]

$$\begin{aligned} 3 &> 2 \\ 3\log_{10}(1/2) &> 2\log_{10}(1/2) \\ \log_{10}(1/2)^3 &> \log_{10}(1/2)^2 \\ (1/2)^3 &> (1/2)^2 \end{aligned}$$

Therefore,

False Theorem 1.1.

$$1/8 > 1/4.$$

(b) You are richer than you think:[2]

[1] Stueben, Michael and Diane Sandford. *Twenty Years Before the Blackboard*, Math. Assoc America, ©1998, p.??.

[2] Stueben, Michael and Diane Sandford. *ibid*, *p.27.*

False Theorem 1.2.

$$1¢ = \$0.01 = (\$0.1)^2 = (10¢)^2 = 100¢ = \$1.$$

(c) Theorem 1.3. *If x is a real number and $(2x-5)/(x-4) = 3$, then $x = 7$.*

False proof. Suppose $x = 7$. Then

$$\frac{2x-5}{x-4} = \frac{2(7)-5}{7-4} = \frac{9}{3} = 3.$$

Thus, if $(2x-5)/(x-4) = 3$, then $x = 7$. □

Problem 2. Rosen, Ex 1.2.8(b)

Problem 3. Rosen, Ex 1.3.14(a,f,n,h)

Problem 4. Rosen, Ex 1.4.14

Problem 5. Rosen, Ex 1.5.26

Problem 6. Rosen, Ex 1.6.12

Problem 7. Prove that

$$\gcd(a, b) = \gcd(b, a - b)$$

for all $a, b \in \mathbb{Z}$. *Hint:* See Rosen, §2.4 Lemma 1.

Problem 8. Rosen, Ex 3.1.18(b)