6.045J/18.400J: Automata, Computability and Complexity Nancy Lynch

Homework 9

Due: May 4, 2005

Readings: Sipser, Section 7.5. Also (optionally) see Garey and Johnson's book, "Computers and Intractability: a Guide to NP-Completeness".

Problem 1: (Sipser 7.20) Let G represent an undirected graph and let

$$\mathsf{SPATH} = \{\langle G, a, b, k\rangle \mid G \text{ contains a simple path of length at } \textbf{most } k \text{ from } a \text{ to } b\}$$

and

$$\mathsf{LPATH} = \{\langle G, a, b, k\rangle \mid G \text{ contains a simple path of length at } \textbf{least } k \text{ from } a \text{ to } b\}$$

1. Show that $\mathsf{SPATH} \in \mathrm{P}$.
2. Show that LPATH is NP-complete. You may assume the NP-completeness of $\mathsf{UHAMPATH}$, the Hamiltonian path problem for undirected graphs.

Problem 2: (Sipser 7.) For a cnf-formula ϕ with m variables and c clauses, show that you can construct in polynomial time an NFA with $O(cm)$ states that accepts all non-satisfying assignments, represented as Boolean strings of length m. Conclude that the problem of minimizing NFAs cannot be done in polynomial time unless $P = NP$.

Problem 3: An edge-cover in a graph $G(V, E)$ is a set of edges $E' \subseteq E$ of G such that each vertex in G is the end-point of at least one of the edges in E'. As a language,

$$\mathsf{EDGE\text{-}COVER} = \{\langle G, k\rangle \mid G \text{ is an undirected graph that has an edge-cover with at most } k \text{ edges}\}.$$

Show that $\mathsf{EDGE\text{-}COVER} \in P$. (Recall the problem $\mathsf{VERTEX\text{-}COVER}$ that we proved NP-complete in the class.)

Problem 4: Suppose there exists a family of bijections $\{f_k\}_{k=1}\infty$ such that f_k maps integers of length k onto integers of length k. We also know that

- For all k, f_k is computable in polynomial time (in k), and
- No f_k^{-1} is computable in polynomial time.

Prove that this would imply that the language

$$A = \{\langle x, y\rangle \mid f^{-1}(x) < y\}$$

is in $(\mathsf{NP} \cap \mathsf{coNP}) \setminus \mathsf{P}$.