6.045J/18.400J: Automata, Computability and Complexity Nancy Lynch

Homework 8

Due: April 20, 2005

Readings: Sections 7.4, 7.5

Problem 1: Let A and B be nontrivial languages over an alphabet Σ (that is, not equal to $\emptyset$ or Σ^*). Explain why each of the following is true.

1. If A is NP-complete, $\overline{A} \in NP$ and $B \in NP$, then $\overline{B}$ must be in NP.
2. If $B \in P$ then $A \cap B \leq_P A$.
3. If $B \in P$ then $A \cup B \leq_P A$.
4. If $A \cup B$ is NP-complete, $A \in NP$ and $B \in P$, then A is NP-complete.

Problem 2: For each of the following pairs of sets A and B, show that $A \leq_P B$.

1. $A = SAT$, and
   $B = TRIPLE-SAT = \{\langle\phi\rangle | \ \phi$ is a satisfiable Boolean formula that has at least three distinct satisfying assignments $\}$.
2. $A = VC$, the Vertex Cover problem, and
   $B = HALF{-}VC$, defined as $\{\langle G\rangle | \ G$ is an undirected graph with an even number of vertices, of which some half form a vertex cover $\}$.

Problem 3: (Sipser 7.39) In the proof of the Cook-Levin theorem, a window is defined to be a 2 by 3 rectangle of cells. Show why the proof would have failed if we had used 2 by 2 windows instead.

Problem 4: (Sipser Problem 7.42) A 2cnf-formula is an AND of clauses, where each clause is an OR of at most two literals. Let $2SAT = \{\langle\phi\rangle \mid \phi$ is a satisfiable 2cnf-formula$\}$. Show that $2SAT \in P$.

Problem 5: (Sipser 7.37) Show that, if P=NP, it is possible to factor positive integers into their prime factors in polynomial time. (Note: NP is a class of *languages* and here, you are being asked for an *algorithm* that produces a factorization for a given integer (as opposed to deciding a language). Thus, simply saying that, "because non-primality is in NP, you are done" isn't enough.)