6.045J/18.400J: Automata, Computability and Complexity Nancy Lynch

Homework 7

Due: April 13, 2005

Readings: Sections 7.1, 7.2, 7.3

Problem 1: Answer each of the following with TRUE or FALSE. You do not need to justify your answers. (Note: when dealing with sets like $O(f(n))$, $\Omega(f(n))$, etc., we use the symbols $=$ and $\in$ interchangeably.)

1. $3 = O(n)$
2. $12n = O(n)$
3. $n^4 = O(n^3 \log^3(n))$
4. $3n \log(n) + 1000n = O(n^2)$
5. $3^n = O(2^n)$
6. $3^n = 2^{O(n)}$
7. $2^{2^n} = O(2^{2^n})$
8. $n^n = O(n!)$
9. $n = o(3n)$
10. $1000n = o(n^3)$
11. $3^n = o(4^n)$
12. $1000 = o(n)$
13. $n = o(\log^2(n))$
14. $\frac{1}{2} = o(1)$
15. $log_2(n) = \Theta(log_{10}(n))$
16. $3^n = \Theta(4^n)$
17. $n^3 = \Theta(8^{log_2(n)})$
18. $n^2 = \Omega(n^3)$
19. $log(n) = \Omega(log(log(n)))$
20. $4^{2^n} = \Omega(2^{4^n})$

Problem 2: (Sipser problem 7.12)
Let

$$MODEXP = \{\langle a, b, c, p\rangle \mid \ a, b, c \text{ and } p \text{ are binary integers such that } a^b \equiv c \text{ (mod p)}\}.$$

Show that $MODEXP$ is in P. (Note that the first and the most obvious algorithm you would come up would run in time *exponential in the input length.* Hint: Try it first when b is a power of 2.)

Problem 3: (Based on Sipser problem 7.14) Prove that P is closed under:

1. The concatenation operation.
2. The star operation.

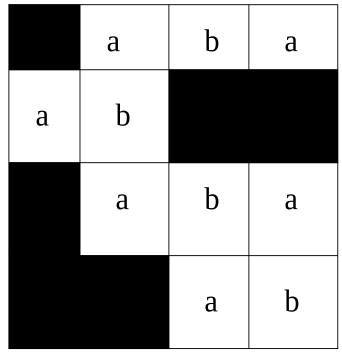
Problem 4: Prove that NP is closed under:

1. The intersection operation.
2. The concatenation operation.

Problem 5: Prove that the following languages are in NP. You may use either the guess-and-check (certificate/verifier) method, or else describe a nondeterministic Turing machine that decides the language in time polynomial in the length of the input.

1	0	0	0
0	0	1	1
1	0	0	0
1	1	0	0

A



B

W = { a, b, ab, ba, aba }

1. (From Sipser exercise 7.11)

   $$ISO = \{\langle G, H\rangle |\text{ G and H are undirected graphs and G and H are isomorphic }\}$$

   (Two graphs are *isomorphic* if, by renaming the nodes of one, we get a graph that is identical to the other.)

2. $TRIPLE\text{-}SAT = \{\langle\phi\rangle | \phi$ is a Boolean formula and ϕ has at least three distinct satisfying assignments $\}$ (Boolean formulas are defined on p. 271 of Sipser's book.)

3. A crossword puzzle construction problem is specified by a finite set $W \subseteq \Sigma^*$ of words, and an $n \times n$ matrix A whose entries are either 0 or 1 (intuitively, a 0 corresponds to a blank square, and a 1 corresponds to a black square). The goal is to use the words in W to fill in the blank squares. Formally, suppose E is the set of all pairs (i, j) such that A_{ij}, the $(i, j)^{th}$ entry of A, is 0. We want to find a mapping $f : E \to \Sigma$ such that the letters assigned to any maximal horizontal or vertical contiguous sequence of members of E form, in order, a word of W. If this is possible, we say that (W, A) is a *constructable crossword system.*

   $$CROSSWORD = \{(W, A) \mid W \subseteq \Sigma^* \text{ and } A \text{ is an } n \times n\ 0-1 \text{ matrix and}$$

   $$(W, A) \text{ is a constructable crossword system.}\}$$

   (For instance, the set $W = \{a, b, ab, ba, aba\}$ over the alphabet $\{0, 1\}$ and the matrix A as in the figure form a constructable crossword system. One of the crosswords so constructed is the matrix B in the figure.)