6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch

Recitation 11: NP-Completeness

April 21, 2005

Readings: Sections 7.4, 7.5

Outline for Today: Lets look back at what we did this week..

1. $SAT = \{\langle\phi\rangle |\ \phi$ is a satisfiable Boolean formula$\}$
2. Cook-Levin Theorem: $SAT \in$ P iff P=NP. That is, SAT is NP-complete. Review proof of Cook-Levin Theorem.
3. What about coNP-completeness ? Show that the complement of SAT is $coNP$-complete. The $NP \stackrel{?}{=} coNP$ question is quite relevant in practice too. Consider the problem of program-checking.

Problem 1: Let $HALF-CLIQUE = \{\langle G\rangle \mid$ G is an undirected graph having a clique of size at least $n/2$, where n is the number of vertices in G $\}$. Show that $HALF-CLIQUE$ is NP-complete. (Build on the $CLIQUE$ problem).

Problem 2:(Sipser 7.29) Show that, if $P = NP$, a polynomial time algorithm exists that, given a Boolean formula ϕ, actually produces a satisfying assignment for ϕ, if it is satisfiable. (Note: NP is a class of *languages* and this problem is the description of a *function*, that takes a formula ϕ and produces a satisfying assignment if ϕ is satisfiable, and a special symbol $\perp$ if it is not.)

If we get time, we will do this fun problem too.
Problem 3:

1. Show that UNARY-PRIMES=$\{1^n \mid n$ is a prime number $\}$ is in P. (Hmm, this is cheating!)
2. Show that PRIMES=$\{n \mid n$ is a prime number in binary$\}$ is in NP and coNP.