6.045J/18.400J: Automata, Computability and Complexity Nancy Lynch

Homework 6

Due: April 6, 2005

Reading: Sipser, Sections 6.1, 6.3

Problem 1: (Counter Machine Decidability) Show that the acceptance problem for 1-counter machines is decidable, by describing a procedure that correctly decides it.
(Hint: Develop a way of detecting situations in which the counter machine must be in a loop.)

For the final two problems, you will have to consider some new definitions that were not covered in lecture, but are presented in Section 6.3 of Sipser's book. Namely, an *Oracle Turing Machine* is a variant of a Turing machine that has the ability to query an external source – an "oracle" – about membership of a string in some particular *oracle language*. A language is *decidable relative to* a language L if it can be decided by an oracle Turing machine that uses L as its oracle set.

Problem 2: (From Sipser Problem 6.19.)
Recall the Post Correspondence Problem discussed in class and in Section 5.2 of Sipser. Show that PCP is decidable relative to A_{TM}, the acceptance problem for ordinary Turing machines.

Problem 3: (From Sipser Exercise 6.4.)
Let $A'_{TM} = \{\langle M, w\rangle |\ M \text{ is an oracle Turing machine and } M^{A_{TM}} \text{ accepts } w\}$. Thus, A'_{TM} can be thought of as the "acceptance problem for oracle TMs relative to the acceptance problem for ordinary TMs".

Show that A'_{TM} is not decidable relative to A_{TM}. That is, even with an oracle for the ordinary acceptance problem, the relative version of the acceptance problem still cannot be decided!

(Hint: Use a diagonalization argument like the one used to prove undecidability of A_{TM}.)