6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch

Homework 2.5 (FAKE)

Due: never

This "fake" homework is intended as a study guide covering the material on lectures 4 and 5.

Problem 1: (Taken from Sipser 1.18.) Give regular expressions generating the following languages. In all cases the alphabet is $\{0, 1\}$.

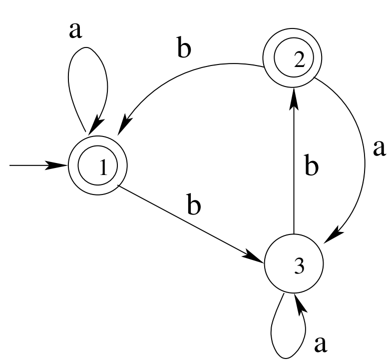
1. $L_1 = \{w|w \text{ contains the substring } 0101\}$.
2. $L_2 = \{w \mid w \text{ does not contain } 100 \text{ as a substring}\}$.
3. $L_3 = \{w|w \text{ starts with } 0 \text{ and has odd length, or starts with } 1 \text{ and has even length }\}$.
4. $L_4 = \{w| \text{ the length of } w \text{ is at most } 5\}$.
5. $L_5 = \{w| \text{ contains at least one } 0 \text{ and at most one } 1 \text{ }\}$.
6. $L_6 = \{w|w \neq \epsilon\}$.

Problem 2: (Sipser 1.19.) Use the procedure described in Lemma 1.29 to convert the following regular expressions to nondeterministic finite automata.

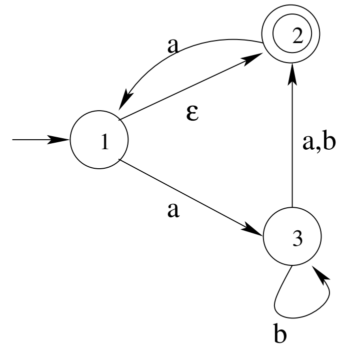
1. $(0 \cup 1)^*000(0 \cup 1)^*$
2. $(((00)^*(11)) \cup 01)^*$
3. $\emptyset^*$

Problem 3: Convert the following finite automata to equivalent regular expressions:

1. The DFA depicted in the following diagram. Use the procedure described in Lemma 1.60.



2. The NFA depicted in the following diagram.



Problem 4: Use the pumping lemma to show that the following languages are not regular.

(a) $A_1 = \{0^a 1^b 2^c | \ 0 \leq a \leq b \leq c\}$.

(b) (From Sipser 1.29.) $A_2 = \{a^{2^n} | \ n \geq 0\}$. (Here, a^{2^n} means a string of 2^n a's.)

(c) $A_3 = \{0^{n^2} \mid n \geq 0\}$.

(d) Do you see something in common between the arguments used to answer parts (b) and (c) ? Generalize the arguments of parts (b) and (c) to show that for any function $f : \mathbb{N} \to \mathbb{N}$ which obeys the inequality $f(n+1) - f(n) > n$, the language $A_4 = \{0^{f(n)} \mid n \geq 0\}$ is not regular.

Problem 5: (Sipser 1.46.) Prove that the following languages are not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersection and complement.

(a) $\{0^n 1^m 0^n \mid m, n \geq 0\}$

(b) $\{w \mid w \in \{0,1\}^*$ is not a palindrome$\}$.[1]

Problem 6: (Sipser 1.53.) Let $\Sigma = \{0, 1, +, =\}$ and

$$ADD = \{x = y + z \mid x, y, z \text{ are binary integers, and x is the sum of y and z } \}$$

Show that ADD is not regular.

[1] A palindrome is a string that reads the same forward and backward. i.e, $w = w^{\mathcal{R}}$.