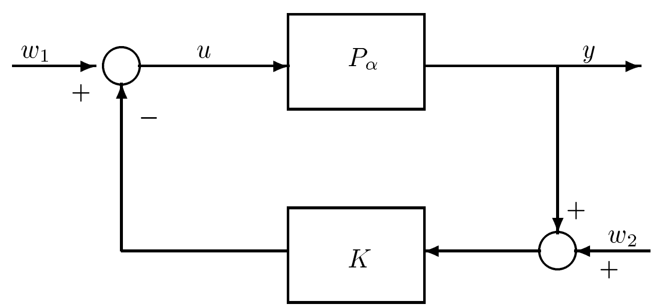
Exercises

Exercise 17.1 Let $P(s) = e^{-2s} - 1$ be connected in a unity feedback configuration. Is this system well-posed?

Exercise 17.2 Assume that P_α and K in the diagram are given by:

$$P_\alpha(s) = \begin{pmatrix} \frac{s}{s+1} & \frac{-\alpha}{s+1} \\ \frac{1}{(s+1)} & \frac{1}{s+1} \end{pmatrix}, \quad \alpha \in \mathbb{R}, \qquad K(s) = \begin{pmatrix} \frac{s+1}{s(s+5)} & 0 \\ -\frac{s+1}{s(s+5)} & \frac{s+1}{s+5} \end{pmatrix}.$$



1. Is the closed loop system stable for all $\alpha > 0$?
2. Is the closed loop system stable for $\alpha = 0$?

Exercise 17.3 Consider the standard servo loop, with

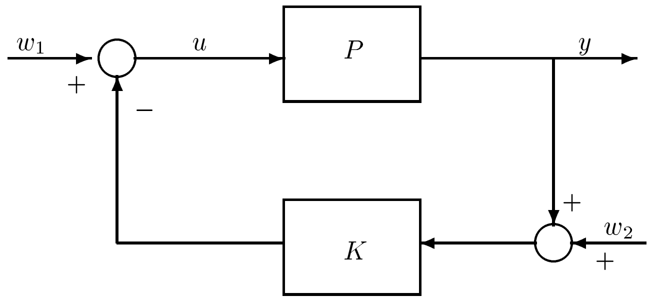
$$P(s) = \frac{1}{10s+1}, \quad K(s) = k$$

but with no measurement noise. Find the least positive gain such that the following are *all* true:

- The feedback system is internally stable.
- With no disturbance at the plant output ($d(t) \equiv 0$), and with a unit step on the command signal $r(t)$, the error $e(t) = r(t) - y(t)$ settles to $|e(\infty)| \leq 0.1$.
- Show that the $\mathcal{L}_2$ to $\mathcal{L}_\infty$ induced norm of a SISO system is given by $\mathcal{H}_2$ norm of the system.
- With zero command ($r(t) \equiv 0$), $\|y\|_\infty \leq 0.1$ for all $d(t)$ such $\|d\|_2 \leq 1$. [ADD NEW Problem]

Exercise 17.4 Parametrization of Stabilizing Controllers

Consider the diagram shown below where P is a given stable plant. We will show a simple way of parametrizing all stabilizing controllers for this plant. The plant as well as the controllers are finite dimensional.



1. Show that the feedback controller

$$K = Q(I - PQ)^{-1} = (I - QP)^{-1}Q$$

for any stable rational Q is a stabilizing controller for the closed loop system.

2. Show that every stabilizing controller is given by $K = Q(I - PQ)^{-1}$ for some stable Q. (Hint: Express Q in terms of P and K).

3. Suppose P is SISO, w_1 is a step, and $w_2 = 0$. What conditions does Q have to satisfy for the steady state value of u to be zero. Is it always possible to satisfy this condition?

Exercise 18.2 Let a plant be given by

$$G(s) = \begin{pmatrix} \frac{s-1}{s+1} & -5 \\ \frac{s+2}{(s+1)^2} & \frac{s-1}{s+1} \end{pmatrix}.$$

We are interested in verifying whether or not there exists a controller K such that the output sensitivity $S = (I + PK)^{-1}$ satisfies $\|S\|_\infty < 1$ (i.e., the maximum singular value is strictly less than 1 for all frequencies). If this is possible, we would like to find such a controller.

1. One engineer argued as follows: Since the transfer functions from u_1 to y_1 and u_2 to y_2 have nonminimum-phase zeros, then the sensitivity cannot be uniformly attenuated. Do you accept this argument. If so, explain her/his rationale, and if not explain why not.

2. Another engineer suggested that the controller can invert the plant and add a scaling factor, so that the sensitivity is uniformly less than 1. Again discuss this option and argue for it or against it.