MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Electrical Engineering and Computer Science

6.302 Feedback Systems

Fall Term 2002 — Issued : November 4, 2002

Prelab 2 — Due : Wednesday, November 13, 2002

Introduction

The purpose of Lab 2 is to give you an opportunity to design a variety of series compensators for a system with well known dynamics. This prelab examines the system to be compensated, as well as the characteristics of three electrical networks which will be used as compensators. You will also analytically determine appropriate compensation strategies and select the proper component values to implement these compensators. In the following lab, you will verify that your compensators work as predicted.

Since this is a very long prelab, we implore you to start early.

Familiarization and Standardization

The heart of our system with well known dynamics will be a "pseudo op-amp," which is represented by the symbol in Figure 1.

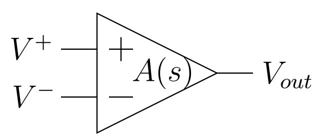

Figure 1: Symbol for pseudo op-amp

The transfer function of the pseudo op-amp is $A(s)$, meaning that $V_{out} = A(s)\,(V^+ - V^-)$. Like a standard op-amp, the pseudo op-amp is assumed to draw negligible current. In the laboratory, you will build a circuit which yields the following transfer function:

$$A(s) = \frac{a_0}{(\tau s + 1)(10^{-3}s + 1)(10^{-4}s + 1)}$$

1. Using Matlab, construct Bode magnitude and phase plots of $A(s)$ for each of the following cases:

   (a) $a_0 = 10^5$, $\tau = 1\text{sec}$

   (b) $a_0 = 10^5$, $\tau = 0.1\text{sec}$

   (c) $a_0 = 10^4$, $\tau = 1\text{sec}$

   (d) $a_0 = 10^4$, $\tau = 0.1\text{sec}$

   Note that the *phase crossover frequency* ω_ϕ (where $\angle A(s) = -180°$) is virtually independent of the values of both a_0 and τ. Additionally, note that the *magnitude* of $A(s)$ at $\omega = \omega_\phi$ is dependent on both a_0 and τ.

2. Consider the circuit in Figure 2.

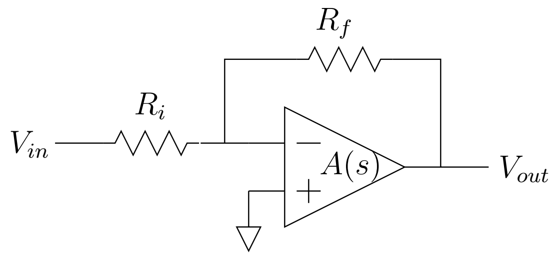

Figure 2: Op-Amp configuration 1

(a) Show that this circuit can be represented by the block diagram in Figure 3.

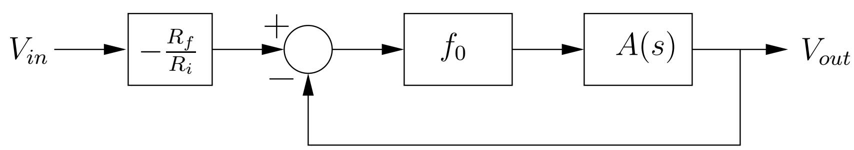

Figure 3: Block diagram configuration 1

where $f_0 = \frac{R_i}{R_i+R_f}$.

(b) You are told that the loop transmission $L(s) = f_0 A(s)$ has a phase margin of $0°$ when $R_i = 62k\Omega$ and $R_f = 220k\Omega$. Find the value of τ in $A(s)$, given $a_0 = 10^5$. (In the lab, you will set τ in this circuit by choosing a capacitor to produce a system on the verge of instability.)

(c) Using your value for τ, find the phase margin when $R_i = 22k\Omega$ and $R_f = 220k\Omega$. What are M_p and ω_n? (These resistor values in the above circuit will make up the uncompensated system in the actual lab exercise.)

Designing Different Compensators

3. For each of the following circuits, draw the block diagram, normalized so that the closed-loop expression $-\frac{R_f}{R_i}$ appears outside of the loop, as in Figure 4.

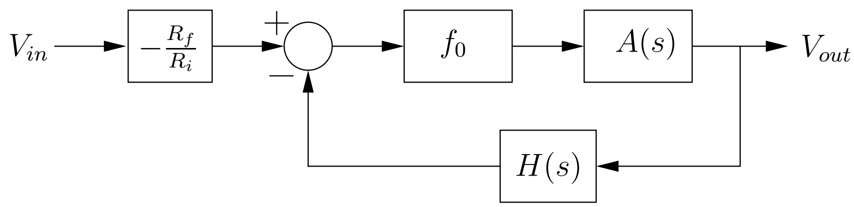

Figure 4: General Block Diagram Form

Note that the loop transmission is $L(s) = G(s)H(s)A(s)$, and that $H(s)$ may be unity.

Find expressions for the appropriate constants (f_0, α, and τ_L) in terms of the component values and write the loop transmissions in the proper form for Figures 5, 7, and 6.

(a) Reduced DC gain:

$$L(s) = A(s)f_0$$

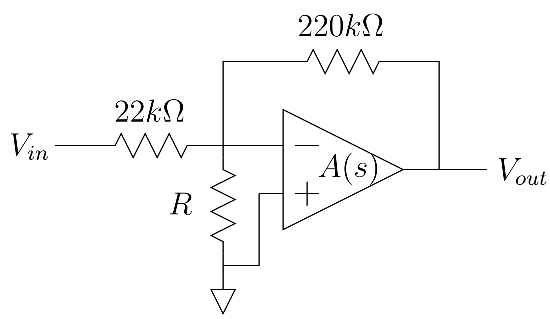

Figure 5: Reduced DC gain

(b) Lag compensation:

$$L(s) = A(s) f_0 \frac{\tau_L s + 1}{\alpha \tau_L s + 1}$$

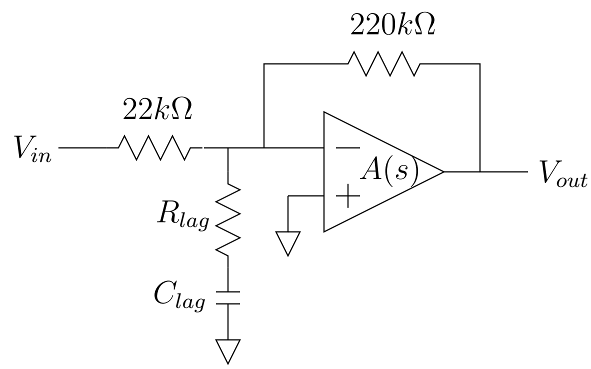

Figure 6: Lag compensation

(c) Lead compensation:

$$L(s) = A(s) f_0 \frac{\alpha \tau_L s + 1}{\tau_L s + 1}$$

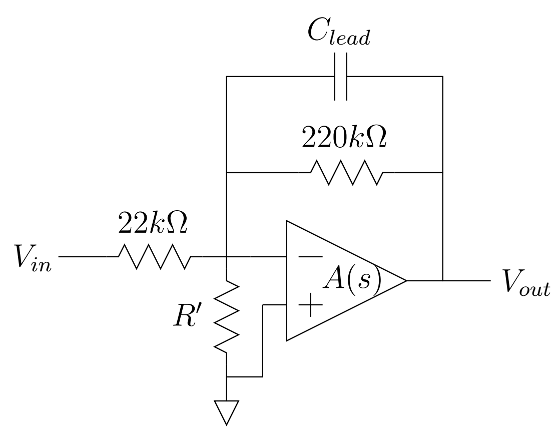

Figure 7: Lead compensation

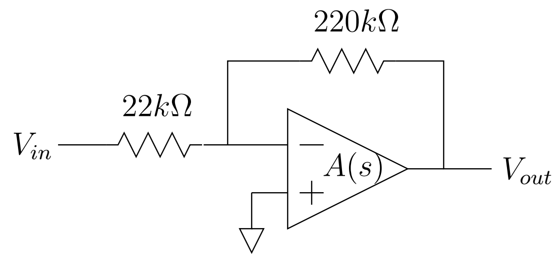

Figure 8: Uncompensated Op-Amp

4. The uncompensated system is shown in Figure 8.

   Note that the loop gain function for the uncompensated system is

$$L(s) = f_0 A(s) = \frac{a_0 f_0}{(\tau s+1)(10^{-3}s+1)(10^{-4}s+1)}; \qquad a_0 = 10^5,\, f_0 = \frac{1}{11},\, \tau \sim 2 \text{ sec} \tag{1}$$

   (a) Compensate the loop gain function to have a phase margin $\phi_{\mathbf{M}} = \mathbf{50}^\circ$ by reducing the DC open-loop gain $a_0 f_0$. Find the value of f_0 which gives the desired phase margin. You should not needlessly sacrifice DC gain or bandwidth.

   (b) Use a lag compensator to achieve a phase margin $\phi_{\mathbf{M}} = \mathbf{44}^\circ$:

$$L(s) = \frac{a_0 f_0}{(\tau s+1)(10^{-3}s+1)(10^{-4}s+1)} \left(\frac{\tau_L s+1}{\alpha \tau_L s+1}\right) \tag{2}$$

   Determine appropriate values for α and τ_L. Again, you should not reduce the bandwidth more than necessary. Note that you may not change f_0.

   Using Matlab, draw root locus plots of the uncompensated and compensated systems, and explain the effect of your lag compensator.

   (c) Compensate the system to have $\phi_{\mathbf{M}} = \mathbf{50}^\circ$ using lead compensation:

$$L(s) = \frac{a_0 f_0}{(\tau s+1)(10^{-3}s+1)(10^{-4}s+1)} \left(\frac{\alpha \tau_L s+1}{\tau_L s+1}\right) \tag{3}$$

   You may not be able to meet the phase margin specification by using lead compensation alone; you may reduce the DC gain margin slightly (by reducing f_0) to meet the design goal. Note that α and f_0 are *not* independent. You should not reduce the DC gain any more than necessary. Find the appropriate values for f_0, α, and τ_L. One strategy might be to maximize your phase margin using $\alpha = 11$, then reduce f_0 (increase α) until you meet your phase margin specification.

   Again, draw root locus plots to demonstrate the effect of your lead compensator.

5. Predicted responses

   (a) For the uncompensated system and each of the compensated systems, determine the:

      i. Crossover frequency (ω_c) and bandwidth (ω_b)
      ii. Dominant closed-loop pole locations (using Matlab)
      iii. Damping ratio (ζ), and natural frequency (ω_n) for 5(a)ii
      iv. Predicted percent overshoot ($P.O.$) and peak time (t_p)
      v. Predicted peak magnitude (M_p) and frequency (ω_p)

      Construct a table showing these values for each system, and make sure that this table comprises the front page of this prelab. Points will be deducted if this is not done.

   (b) For the uncompensated system and each of the compensated systems, use Matlab to plot:

      i. The open-loop Bode plot
      ii. The closed-loop Bode magnitude response
      iii. The closed-loop unit step response

6. Component values

   By using the transfer functions found for each of the compensation circuits, determine the necessary values for each of the circuit components in order to successfully implement the compensators designed above:

   (a) R for the reduced gain compensator
   (b) R_{lag} and C_{lag} for the lag compensator
   (c) R' and C_{lead} for the lead compensator

   Make sure that you enter these values in the table at the front of the prelab as well.