

6.302 Feedback Systems

Fall Term 2002
Problem Set 5

Issued : October 7, 2002
Due : Monday, October 21, 2002

Problem 1: Experimental Nyquist

As has been pointed out in class and in the notes, an advantage of the Nyquist criterion over the Routh test or root locus is that you don't need a rational transfer function to study stability. This can come in handy when, given frequency response data for an open loop system $L(s)$, you want to learn about stability issues that may arise when closing a feedback loop around that $L(s)$. One such set of data is given in Figure 1; using this plot, sketch the Nyquist locus for $L(s)$ under the assumption that you are using unity feedback.

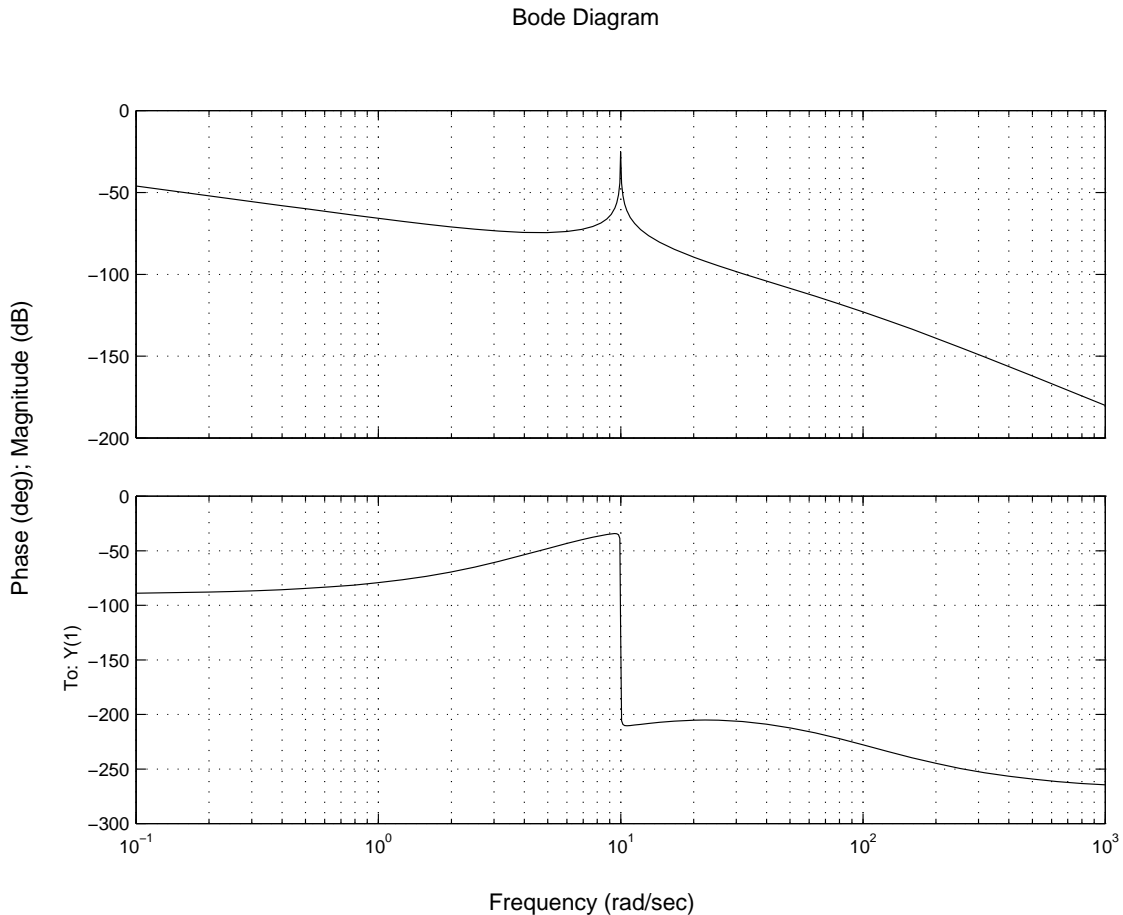


Figure 1: Frequency Response of $L(s)$

Problem 2: Relating Characteristics of the Open Loop to the Closed Loop

1. For a unity feedback system, we know that

$$L(s) = \frac{0.25s}{(s+1)(0.01s+1)^3} \quad (18)$$

Using asymptotic approximate methods, find the closed-loop magnitude curve of the Bode diagram for $\frac{C(j\omega)}{R(j\omega)}$.

2. Repeat the first part for

$$L(s) = \frac{K(s+1)}{s^2(0.2s+1)(0.01s+1)} \quad (19)$$

where $K = 50$ dB.

Computer Project 5: Nyquist

This computer project should be completed using Octave, MATLAB or similar software. You may find it helpful to save your work as it may be useful for future projects. Please hand in clearly labelled printouts. The purpose of this project is to investigate the Nyquist stability criterion.

1. Produce Nyquist plots for the following transfer functions. Assume $K = 1$ unless otherwise stated. Produce a table with the range of K for which each system is stable.

$$L_1(s) = \frac{K}{s+1}$$

$$L_2(s) = \frac{K}{(s+1)^2}$$

$$L_3(s) = \frac{K}{(s+1)^3}$$

- (a) Mark on the plot the number of encirclements in each region. Do these numbers agree with the root-locus plot?
- (b) For what positive value of gain K will the system become unstable? This value is known as the gain margin.
- (c) Produce a plot with this value of K on the same axis. Do the same for the negative value of K for which the system becomes unstable.
- (d) By drawing a unit circle around the origin (on the computer, or on a printout using a pair of compasses) find the angle between the real axis and the first point that the plot intersects the unit circle. This angle is known as the phase margin.

$$L_4(s) = \frac{K}{(s^2+2s+2)}$$

Comparing L_4 to L_2 , by what factor must the gain of L_2 be greater than the gain of L_4 for the pole locations of both systems to be identical? How is this reflected in the range of K for which both systems are stable?

$$L_5(s) = \frac{K}{s(s^2 + 2s + 2)}$$

$$L_6(s) = \frac{K(s + 2)}{s(s^2 + 4s + 8)(s + 100)}$$

Note that although you cannot encircle the $-1/K$ point by increasing positive gain, relatively little phase needs to be added to cause an encirclement. What is the phase margin?

$$L_7(s) = \frac{K(s - 1)^2}{(s + 1)^3 * (s + 100)^2}$$

Computer programs often scale the plots to include the -1 point. This often requires the user to zoom in around the origin.

$$L_8(s) = \frac{K(s - 1)}{(s + 2)^2(s + 5)}$$

Sketch the following Nyquist plot by hand and then produce a plot on the computer.

$$L_9(s) = \frac{K}{s^2 + 1}$$

Always double check Nyquist plots produced on a computer if there are poles on the $j\omega$ -axis.

2. Compensating the Triple Integrator.

Reminder: The transfer function of the open loop was

$$L(s) = K \frac{(s + 2)^2}{(s + 100)^2} \frac{1}{s^3}$$

- (a) You show the system ($K = 2.5 \times 10^5$) to your UROP supervisor who lowers the gain only to find that the system becomes unstable. Plot the step response of the closed loop system with a gain $K = 1000$.
- (b) He suggests producing the Nyquist plot to investigate. He asks you to submit:
 - i. An appropriately scaled Nyquist plot
 - ii. As few plots as are necessary to demonstrate the range of K for which the system is stable. (You will need to zoom in)
 - iii. A cartoon sketch of the Nyquist plot which shows all the features including any parts omitted by the software. Label the limits and the number of encirclements for each region.