6.441 Transmission of Information
Problem Set 8

Spring 2003

Due date: April 24

Problem 1, Binary detection Consider an AWGN channel $Y_i = X_i + W_i$, with $W_i \sim N(0, \sigma^2)$ and power constraint $\frac{1}{n}\sum_i x_i^2 \leq P$.

a) Use Matlab, plot the capacity as a function of the signal to noise ratio P/σ^2.

b) Consider a sub-optimal strategy as follows: at each time, transmit X_i as $+\sqrt{P}$ or $-\sqrt{P}$. At the receiver, use maximum likelihood detection to detect whether $+$ or $-$ is transmitted. Compute the probability of detection error P_e.

c) Now each time gives a binary symmetric channel with cross over probability of P_e, compute the capacity of this channel. Argue that we can achieve this capacity by using a binary code to choose $+$ or $-$ to be transmitted at each time. Plot the capacity as a function of the signal-to-noise ratio, and compare with the result in part a)

d) Use the same binary input at each time, but do not assume a hard decision to be made at each time. Give an expression of the capacity. How would this capacity compare with the results in a) and c)?

Problem 2 Consider the following channel

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}$$

where W_1, W_2 are i.i.d. $N(0, \sigma^2)$ additive Gaussian noise. X_1 and X_2 are the transmitted signals of two users. Let X_1 and X_2 be independently Gaussian distributed, with 0 mean and variances P_1, P_2, respectively. The goal of this problem is to communicate with X_1, treating X_2 as interference.

a) Compute $I(X_1; Y)$.

b) As a sub-optimal approach, the receiver first form $R = Y_1 + Y_2$, compute $I(X_1; R)$. Argue that R is not a sufficient statistics. Notice that in the absence of X_2, this R is a sufficient statistics.

c) As another sub-optimal approach, the receiver form $R = Y_2$, compute $I(X_1; R)$. Find an example of values of P_1 and P_2 such that this mutual information is larger than that in part b).

d) Let R be the minimum mean square estimate of X_1 given Y_1, Y_2. Compute $I(X_1; R)$. Is this R a sufficient statistics?

Problem 3 Consider a channel that takes 2-dimensional vectors $\underline{X}$ as input. The channel randomly perturbs the angle of $\underline{X}$ by an amount of Δ, which is uniformly distributed in $[0, \pi/10)$, call the resulting vector $\underline{\tilde{X}}$, and the output of the channel is

$$\underline{Y} = \frac{\underline{\tilde{X}}}{\|\underline{\tilde{X}}\|}$$

Compute the channel capacity. Argue that the i.i.d. Gaussian distribution achieves the capacity. Find appropriate coordinate systems to write the differential entropies of $Y, Y|X, X, X|Y$.