6.441 Transmission of Information
Problem Set 9

Spring 2003

Due date: May 1

Problem 1 The capacity of an additive-noise waveform channel is given by the water-filling formulas

$$C = \frac{1}{2}\int_{-\infty}^{\infty} \max\left[0, \log\frac{\theta}{N(f)}\right] df$$

where $N(f)$ is the power spectral density of the noise process, and θ satisfies

$$P = \int_{-\infty}^{\infty} \max\left[0, \theta - N(f)\right] df$$

a) Understand the formulas, what are the units of all the terms?

b) Consider a jammer that wants to design a $N(f)$ to minimize the capacity, subject to a power constraint

$$J = \int N(f) df$$

and noise is independent of the signal. What noise should he choose? Does it have to be Gaussian?

Problem 2 This problem is covered as an example in the class, you are asked to use Matlab to come up with a similar result as the one shown in class. You will need no more than 20 lines of code. The purpose is to allow you try a few other things on your own.

Consider a colored Gaussian noise channel

$$\underline{Y} = \underline{X} + \underline{W}$$

where $\underline{X}, \underline{W}, \underline{Y}$ are all two dimensional vectors. $\underline{W}$ has zero mean and covariance matrix

$$K_W = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$$

Let the power constraint be $E[\|\underline{X}\|^2] \leq 10$. Notice the feedback in this case is very simple

$$\begin{aligned} X_1 &= V_1 \\ X_2 &= bW_1 + V_2 \end{aligned}$$

where $\underline{V}$ is independent of $\underline{W}$.

Use Matlab to compute the achievable data rate as a function of b, (find the right power constraint on $\underline{V}$, and use water-filling). Find the optimal feedback value b.