# 6.441 Transmission of Information
# Problem Set 6

Spring 2003

Due date: April 10

**Problem 1**
Given two DMC's with capacities $C_1$ and $C_2$, we construct the following system. Let the source $V$ be observed by encoder 1, the coded symbols are then passed through channel 1; the output of channel 1 is feed into encoder 2, which transmit coded symbols over channel 2; the output of channel 2 is decoded to recover the source $V$; this output is also feedback to encoder 1 in a causal fashion. We are allowed to design both encoder 1 and 2, as well as the decoder.

Show that the capacity of this system is $\min\{C_1, C_2\}$.

**Problem 2** Consider a binary asymmetric channel as follows:

$$\begin{aligned} P_{Y|X}(1|0) &= \epsilon_1 \\ P_{Y|X}(0|1) &= \epsilon_2 \end{aligned}$$

a) Write $H(Y)$ and $H(Y|X)$ in terms of $H(\epsilon_1)$ and $H(\epsilon_2)$, where $H(\epsilon) = -\epsilon \log \epsilon - (1 - \epsilon) \log(1 - \epsilon)$.

b) Draw a graph of $H(\epsilon)$, read the capacity of this channel from the graph. (you don't need to give explicit solution of the optimal input distribution, but only need to find that on the graph)

c) Is your result consistent with the case $\epsilon_1 = \epsilon_2$?

d) Notice for $\epsilon_1 \neq \epsilon_2$, the optimal input distribution is not equiprobable. Explain what redundancy is added by the channel coder.

**Problem 3** Consider a BSC with the crossover probability $\epsilon$ being random too. Let $\epsilon = \epsilon_1$ with probability 1/2, and $\epsilon = \epsilon_2$ with the probability of another half. Assume $0 \leq \epsilon_1 \leq \epsilon_2 \leq 1/2$.

a) Assume that $\epsilon$ changes independently in each channel use according to the above distribution, and that the actual value of $\epsilon$ is unknown to the transmitter, what is the channel capacity?

b) Assume now $\epsilon$, although random, remain fixed for a block of $T$ channel uses, before changing into independent realizations in the next block. Assume $T$ is large enough. Now during the first $T_1$ channel uses in each block, $1 << T_1 << T$, the receiver can actually estimate the channel parameter $\epsilon$ precisely, and feedback to the transmitter. Compute the channel capacity with such feedback (you are only asked to compute the asymptotic capacity with $1 << T_1 << T$).

c) Conclude that with the feedback, the capacity of the channel increases, and explain why this does not violate the feedback capacity theorem.