6.441 Transmission of Information
Problem Set 5

Spring 2003

Due date: March 20

Problem 1
For any given distributions $P_X(x)$ and $P_{Y|X}(y|x)$, prove that

$$I(X;Y) = \min_{\widehat{P}_Y} \sum_{y \in \mathcal{Y}} \sum_{x \in \mathcal{X}} P_X(x) P_{Y|X}(y|x) \log \left(\frac{P_{Y|X}(y|x)}{\widehat{P}_Y(y)} \right)$$

where the minimization is taken over all distributions over $\mathcal{Y}$.

Problem 2 Let U, V, X, Y, and Z be random variables, do the conditions

$$I(V; Y, Z|U, X) = 0$$

and

$$I(X; U, Z|V, Y) = 0$$

imply that

$$I(Z; X, V|U, Y) = 0?$$

Prove if true, or give a counter example if false.

Problem 3 Consider a quaternary erasure channel as follows: the input alphabet is $\mathcal{X} = \{1, 2, 3, 4\}$ and the output alphabet is $\mathcal{Y} = \{1, 2, 3, 4, E\}$. The transition probability is

$$\begin{aligned} P(Y = i|X = i) &= 1 - \epsilon \\ P(Y = E|X = i) &= \epsilon \end{aligned}$$

for $i = 1, 2, 3, 4$.
a) Is this a symmetric channel?
b) Compute the channel capacity?
c) Compare your result with the capacity of two independent parallel binary erasure channels, explain your result intuitively.