

6.441 Transmission of Information

Problem Set 4

Spring 2003

Due date: March 13

Problem 1

Problem 8.8 in Cover and Thomas.

Problem 2

Problem 8.9 in Cover and Thomas.

Problem 3

Problem 8.12 in Cover and Thomas.

Problem 4 Channel Capacity with Memory

Consider a binary symmetric channel with $Y_i = X_i \oplus Z_i$, where \oplus is the mod 2 addition. and $X, Y \in \{0, 1\}$. Assume \underline{Z}^n is independent of \underline{X}^n . Suppose $\{Z_i\}$ has constant marginal probabilities $P(Z_i = 1) = p = 1 - P(Z_i = 0)$, but Z_1, Z_2, \dots, Z_n are not necessarily independent. That is, the channel has memory, then the capacity

$$C^{(n)} = \max_{P_{\underline{Z}^n}} \frac{1}{n} I(X_1, \dots, X_n; Y_1, \dots, Y_n)$$

can be larger than $(1 - H(p))$, which is the capacity of the memoryless channel. Now consider the extreme case that $Z_1 = Z_2 = \dots = Z_n$, (but still random with the same marginal distribution). Compute the channel capacity, and compare with the capacity for the memoryless case.

Problem 5 Independent Parallel Channels

Let C_1 and C_2 be the capacity of two DMC's $P_1(y_1|x_1)$ and $P_2(y_2|x_2)$, respectively. Determine the capacity of the DMC

$$P(y_1, y_2|x_1, x_2) = P_1(y_1|x_1)P_2(y_2|x_2)$$

Interpret your result intuitively.