6.441 Transmission of Information
Problem Set 10

Spring 2003

Due date: May 8

Problem 1 A multiple access channel has two binary inputs and a ternary output with symbols $\{0, 1, e\}$. The channel outputs 0 if both inputs are 0, and 1 if both inputs are 1, and e otherwise. Compute the capacity region.

Problem 2 A Gaussian multiple access channel with M users, each with power P,

$$Y = \sum_{i=1}^{M} X_i + W$$

where W is bandlimited white noise with variance N, and all inputs are constrained to lie in a common band of of WHz.

a) What is the capacity region?

b) Assume $R_i = R$ for all i. Let $P = E_b R, N = N_0 W$, and $r = MR/W$, show that

$$\frac{E_b}{N_0} > \frac{2^r - 1}{r}$$

Compare this with point-to-point Gaussian channel, what can be concluded about the energy efficiency in multiple access channel?

Problem 3 Consider a 2-dimensional Gaussian multiple access channel,

$$\underline{Y} = \underline{h}_1 X_1 + \underline{h}_2 X_2 + \underline{W}$$

where $\underline{h}_1, \underline{h}_2$ are fixed known 2-D vectors with unit norm, $\underline{W}$ is additive white Gaussian noise with $N(0, \sigma^2)$ entries. X_1, X_2 have power constraints P_1, P_2 respectively. Let $\langle \underline{h}_1, \underline{h}_2 \rangle = a^2$. Assume that $P_1(1 - 2a^2) = P_2$. Assume the users choose the Gaussian input distribution with their full power, i.e., $X_i \sim N(0, P_i) i = 1, 2$.

a) Assume successive cancellation decoding is used. For both decoding orders, compute the achievable rate for both users.

b) Assume that user 1 is decoded first. Now we allow user 1's signal to be an arbitrary 2-D vector, i.e, replace $\underline{h}_1 X_1$ by $\underline{X}_1$, while user 2's signal is unchanged. What is the optimal input distribution for user 1? Compare with your results in part a), and discuss the difference between these two channels.