6.441 Transmission of Information
Problem Set 2

Spring 2003

Due date: February 20

Problem 1 A sender transmits a random variable X through a channel with output Y given by

$$Y = X + Z \bmod 2$$

where Z is the signal inserted by a jammer. X and Z are independent. X and Z can only take binary values 0 or 1. The sender is limited in energy so that $E_X[X^2] \leq \frac{1}{2}$ and the jammer is limited in energy so that $E_X[X^2] \leq \frac{1}{4}$. The sender seeks to maximize mutual information ($I(X;Y)$, while the jammer seeks to minimize it. What is the policy followed by the sender, by the jammer, and the resulting mutual information?

Problem 2
Problem 3.3 in Cover and Thomas.

Problem 3
Problem 3.7 in Cover and Thomas.

Problem 4
Problem 4.1 in Cover and Thomas.

Problem 5
Problem 4.2 in Cover and Thomas.

Problem 6
Problem 4.12 in Cover and Thomas.

Problem 7, Chernoff bound
Use Chebyshev's Inequality to show the following: let $X_1, \ldots, X_n$ be i.i.d. random variables, and $Y = \sum_{i=1}^{n} X_i$.

$$P\left(\frac{Y}{n} \geq A\right) \leq e^{-nsA}\left(E_X[e^{sX}]\right)^n$$

for any $s \geq 0$.