

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF COMPUTER SCIENCE AND ELECTRICAL ENGINEERING

6.801/6.866 MACHINE VISION

Handed out: 2004 Sep. 9th

Due on: 2004 Sep. 16th

Problem 1: In class we solved the least squares problem of estimating the speed at which an image moves in a *one dimensional* camera, based on the constant brightness assumption. The constant brightness assumption (in one dimension) can be expressed in the form

$$uE_x(x, t) + E_t(x, t) = 0,$$

where E_x and E_t are the x and t derivatives of image brightness $E(x, t)$, while $u = dx/dt$ is the speed of the image motion.

Now suppose that the illumination is *not* constant, but changing in such a way that $uE_x + E_t = k$ (rather than being equal to zero). Clearly this is equivalent to using the incorrect value $E'_t = E_t - k$ for the time derivative of the brightness in the original equation. Suppose we do not know the value of k ahead of time.

Let us first blindly apply the least squares method for estimating u . Suppose that the “image” runs from x_1 (with brightness E_1) to x_2 (with brightness E_2).

- (a) What is the least squares solution for u we would get based on the “corrupted” time derivative (E'_t) of brightness?
- (b) Is there any error in the estimated speed u if the brightness at the right end of the image matches that at the left end?

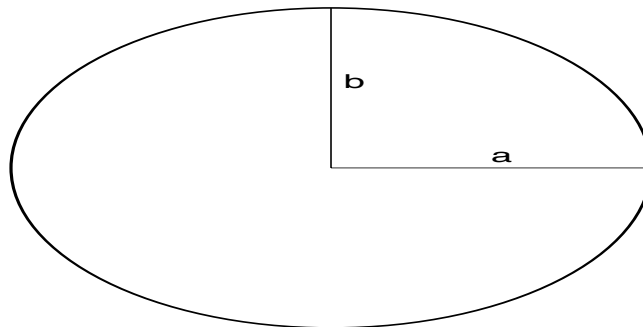
Now let us try to take into account the change of overall illumination with time. We do not know the magnitude of k . We will try and estimate both u and k using a least squares method.

- (c) What are the best fit values for u and k ? Assume that the linear “image” runs from x_1 (with brightness E_1) to x_2 (with brightness E_2).
- (d) How do the expressions for u and k simplify if the brightness at the right end of the image matches that at the left end?

Problem 2: Consider a monocular system for estimating the orientation of a planar surface based on an image of a circle drawn on that surface. If we view the plane “straight on”, the circle will be imaged as a circle (by “straight on” we mean that the optical axis is perpendicular to the plane, and the optical axis passes through the center of the circle). We get an ellipse in the image if the plane is tilted so that its normal vector is no longer parallel to the image plane. From the eccentricity of the ellipse we plan to determine how much the plane is tilted.

To simplify matters, we here consider *orthographic* instead of *perspective* projection. This corresponds to the limit of looking at the objects from further and further away, but with correspondingly higher and higher magnification. Alternatively, we can think of projection along parallel lines as opposed to projection along rays diverging from a center of projection.

Imagine parallel rays perpendicular to the image plane arriving from a very distant light source. A circular disk is suspended above the image plane, blocking some of the incident light. The result is an elliptical shadow in the image. The angle between the image plane and the plane containing the circular disk is θ .

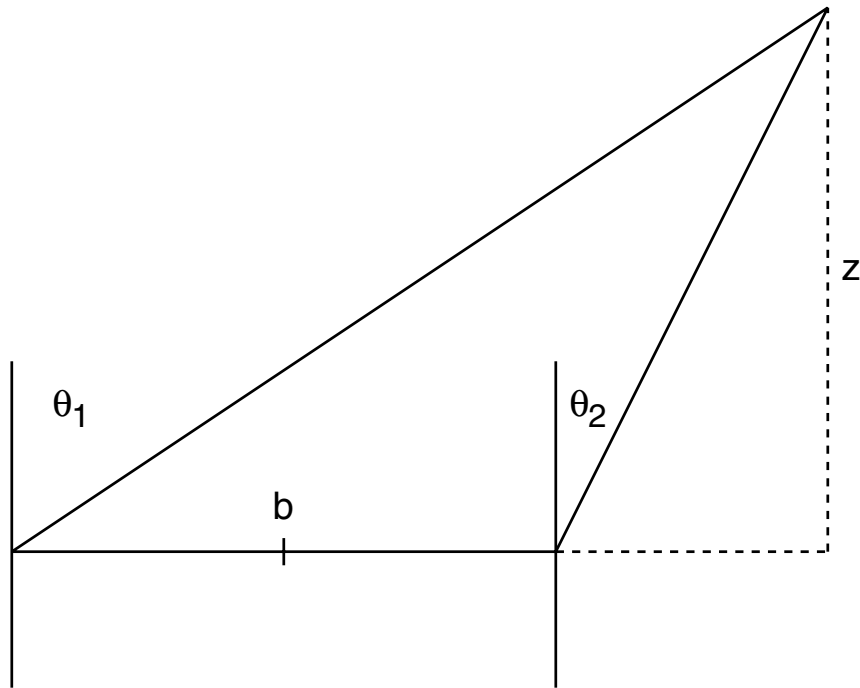


- (a) Find the length of the minor axis b in terms of the angle θ and the length of the major axis a .

The eccentricity e of an ellipse can be defined by $b^2 = a^2(1 - e^2)$. Express the eccentricity of the ellipse in the image in terms of the angle θ .

- (b) Image measurements are hard to make accurately. If we are to estimate θ based on the ratio of b to a , then we should also consider the effect of small errors in the measurement of b (assume a is known accurately). What is the relationship between small errors in measurement δb and corresponding errors in the estimated orientation of the plane $\delta\theta$? Give an expression for the error sensitivity $d\theta/db$ as a function of the eccentricity e of the ellipse.
- (c) If you had to select a position for a camera in a monocular visual wheel alignment system based on imaging the circular rim of the wheel, would you put the camera along the line passing through the axis of the wheel?

Problem 3: A simple *planar* binocular stereo system has a baseline of length b and two ‘cameras’ measuring the angles θ_1 and θ_2 between rays to objects in the world and the baseline. We erect a coordinate system with the x -axis lined up with the baseline and the origin at the midpoint of the baseline. The z -axis is perpendicular to the x -axis.



- (a) Show that the intersection of the two rays lies at

$$x = \frac{b \sin(\theta_2 + \theta_1)}{2 \sin(\theta_1 - \theta_2)} \quad \text{and} \quad z = b \frac{\cos \theta_1 \cos \theta_2}{\sin(\theta_1 - \theta_2)}$$

(also check the equation for x in the two special cases $\theta_2 = 0$ and $\theta_1 = 0$).

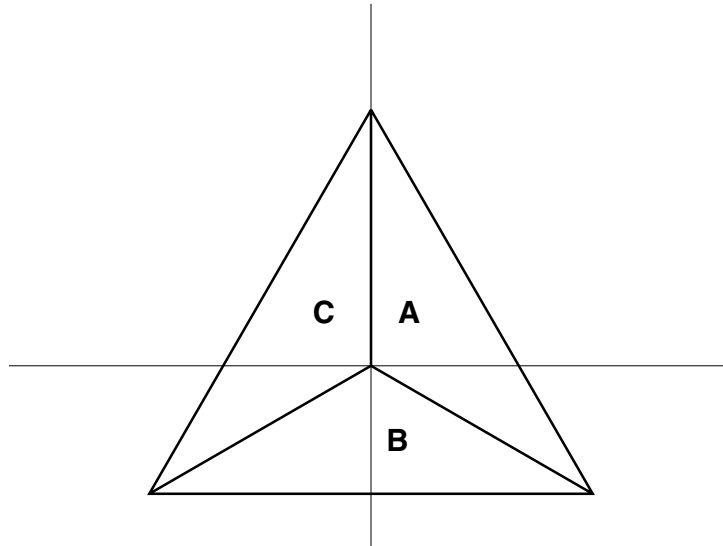
- (b) Show that

$$z \approx b \frac{1}{\sin(\theta_2 - \theta_1)}$$

when $\theta_1 \approx 0$ and $\theta_2 \approx 0$. Hence depth (z) is approximately inversely proportional to disparity ($\theta_1 - \theta_2$) and proportional to the baseline b .

- (c) Measurements of ray directions are never perfectly accurate. Estimate the errors in z resulting from errors δ_1 in measuring θ_1 , and δ_2 in measuring θ_2 . (based on the approximate formula for z derived in part (b)). Show that the absolute error in estimating depth grows as z^2/b . Consequently the relative error (i.e. $\delta z/z$) grows as z/b (Hint: differentiate w.r.t. the angles).

Problem 4: Consider a pyramid of unknown height h with an equilateral base in the x - y plane as shown:



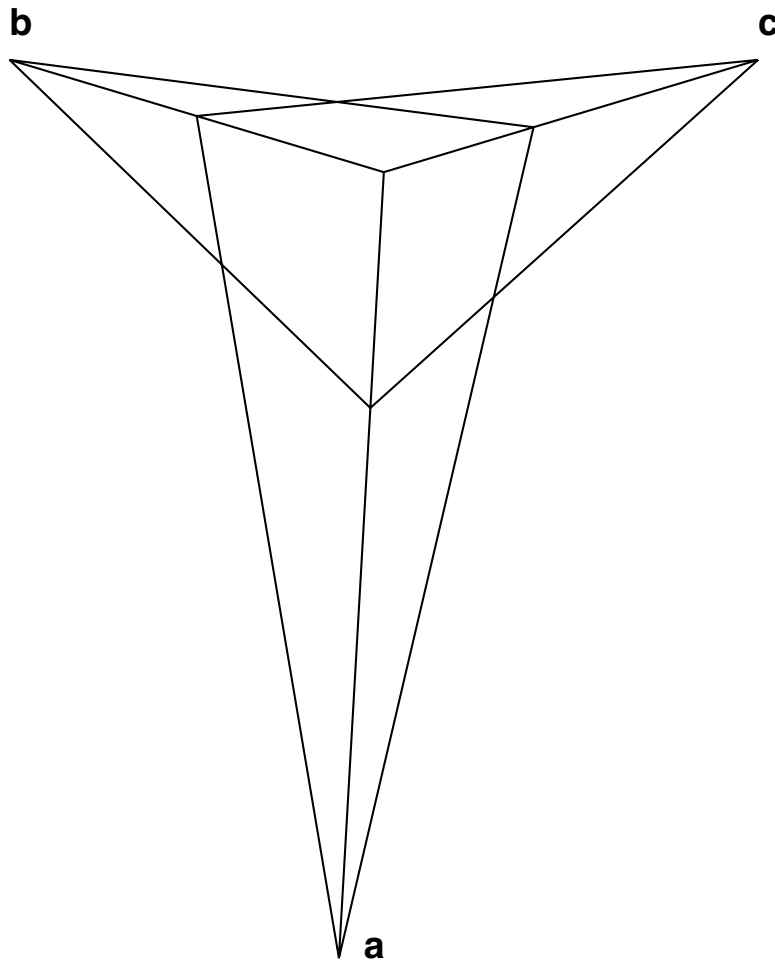
One edge of the base is parallel to the x -axis. The three isosceles triangles forming the sides of the pyramid are coated with matte white paint that obeys Lambert's 'law' and are illuminated by a distant light source of unknown brightness lying in direction $(1/\sqrt{2})(1, 0, 1)^T$. Now suppose that the observed brightnesses of the three facets are in the ratio

$$E_A : E_B : E_C = 3 : 2 : 1$$

What is the height h of the pyramid as a multiple of the length of an edge of the base?

Problem 5: Consider a perspective image of a rectangular brick. Three vanishing points can be obtained by intersecting extended image lines corresponding to parallel edges on the object (see Figure).

- (a) Suppose that the vanishing points are at \mathbf{a} , \mathbf{b} , and \mathbf{c} in the image plane. Suppose that the center of projection is at \mathbf{r} , above the image plane. Write down three (quadratic) equations involving the vectors \mathbf{a} , \mathbf{b} , \mathbf{c} , and \mathbf{r} , based on the expected angles between lines from the center of projection to the vanishing points in the image plane.
- (b) Eliminate quadratic terms to obtain three (redundant) linear equations in \mathbf{r} .



- (c) Consider the triangle formed in the image plane by connecting the vanishing points. Show that the principal point \mathbf{r}' (foot of the perpendicular from the center of projection onto the image plane) lies at the intersection of the perpendiculars dropped from the three vertices onto the opposite sides. (Hint: $\mathbf{r} = \mathbf{r}' + f\hat{\mathbf{z}}$ where f is the principal distance, that is, the height of the center of projection above the image plane, and $\hat{\mathbf{z}}$ is a unit vector perpendicular to the image plane).
- (d) Give a (non-trivial) bound on the principal distance in terms of the maximum or minimum of the lengths of the sides of the triangle.
- (e) Consider the special case when the vanishing points form an equilateral triangle. Show that the principal distance f equals kl , where l is the length of the sides. What is the value of k ?