Massachusetts Institute of Technology

Department of Computer Science and Electrical Engineering

6.801/6.866 Machine Vision

Handed out: 2004 Oct. 7th **Due on: 2004 Oct. 14th**

Problem 1: Do Problem 11-5 in *Robot Vision* (Shape from shading ambiguity).

Problem 2: Do Problem 11-7 in *Robot Vision* (Importance of singular point).

Problem 3: Do Problem 11-16 a–c in *Robot Vision* (Recovering source direction).

Problem 4: (Unknown normals and sources) Let's try and use photometric stereo methods to recover surface orientation *and* light source direction from multiple images.

Suppose we have N planar surface facets with unit normals $\hat{\mathbf{n}}_i$, and M light source directions given by the unit vectors $\hat{\mathbf{s}}_j$. Define $\mathbf{n}_i = \rho_i \hat{\mathbf{n}}_i$ and $\mathbf{s}_j = I_j \hat{\mathbf{s}}_j$, where ρ_i is the albedo of the i-th facet and I_j is the intensity of the j-th source. The brightness recorded in the j-th image of the i-th facet then can be written

$$E_{ij} = \mathbf{n}_j \cdot \mathbf{s}_j$$

With N facets and M light source directions we can gather NM brightness values to try and recover the $3N + 3M$ unknown parameters, provided N and M are large enough (i.e. $NM > 3(N + M)$).

(a) Assume that there is a set of normals $\{\mathbf{n}_i\}$ and a set of source directions $\{\mathbf{s}_j\}$ that satisfy the observed image brightness measurements. Suppose now that we transform the surface normals using a linear transformation as follows $\mathbf{n}'_i = A\mathbf{n}_i$, where A is a 3×3 matrix. Show that there is a linear transformation on the light source vectors yielding new vectors $\mathbf{s}'_j$ such that

$$E_{ij} = \mathbf{n}'_i \cdot \mathbf{s}'_j$$

What are the constraints on A for this construction of alternate solutions to work?

(b) Now suppose that all the patches have the same albedo so that we can treat the normal vectors as unit vectors. That is, replace $\mathbf{n}_i$ in the expression for E_{ij} with $\hat{\mathbf{n}}_i$. What are the constraints now on A? What does this mean in terms of uniqueness or lack thereof?

Problem 5: (Directional Second Derivatives) The brightness gradient $\nabla E = (E_x, E_y)$ is useful in recovering image motion, as well as in "edge" detection. The *second* derivatives of brightness are of use in recovering local surface curvature, as well as in "line" detection.

If $\nabla E = (E_x, E_y)$ is the brightness gradient, then $dE/ds = E_x \cos\theta + E_y \sin\theta$ is the first directional derivative of brightness along a line that makes an angle θ with the x axis. This formula can be used to show that the maximum directional derivative is in the direction given by the unit gradient vector

$$\frac{1}{\sqrt{E_x^2 + E_y^2}}(E_x, E_y)$$

and that the slope in that direction is $\sqrt{E_x^2 + E_y^2}$.

(a) Show that the *second* directional derivative along a line that makes an angle θ with the x axis is:

$$\frac{d^2E}{ds^2} = E_{xx} \cos^2\theta + 2E_{xy} \sin\theta\cos\theta + E_{yy} \sin^2\theta.$$

(b) Show that the second directional derivative has extrema for

$$\tan 2\theta = \frac{2\,E_{xy}}{E_{xx} - E_{yy}}$$

(c) Show that the extreme values are

$$E''_{\min,\max} = \frac{1}{2}(E_{xx} + E_{yy}) \pm \frac{1}{2}\sqrt{(E_{xx} - E_{yy})^2 + 4\,E_{xy}^2}.$$

d) Let

$$D = \sqrt{(E_{xx} - E_{yy})^2 + 4E_{xy}^2}.$$

Show that the four directions of extrema in second directional derivative are given by:

$$\pm\left(\sqrt{\frac{D \pm (E_{xx} - E_{yy})}{2D}}, \operatorname{sign}(E_{xy})\sqrt{\frac{D \mp (E_{xx} - E_{yy})}{2D}}\right).$$

(e) When will there *not* be extrema in the directional second derivative of brightness?

(f) What is the geometric relationship between the gradient direction $\nabla E = (E_x, E_y)$ and the direction of the minimum second directional derivative on 'top of a ridge' of the brightness surface? What is the geometric relationship between the gradient direction and the direction of the maximum second directional derivative at the 'bottom of a valley' of the brightness surface?

Hints: Use identities for trigonometric functions of doubled angles. Express $\sin 2\theta$ and $\cos 2\theta$ in terms of $\tan 2\theta$. Substitute back into the expression for the directional second derivative. Express $\cos\theta$ and $\sin\theta$ in terms of $\cos 2\theta$ and $\sin 2\theta$. Note that the second directional derivative can be positive or negative.