MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF COMPUTER SCIENCE AND ELECTRICAL ENGINEERING

6.801/6.866 MACHINE VISION

Handed out: 2004 Nov 4th **Due on: 2004 Nov 12th**

Problem 1: In the continuous version of the optical flow problem, we find the functions $u(x, y)$ and $v(x, y)$ that minimize

$$\iint_D (uE_x + vE_y + E_t)^2 + \lambda(u_x^2 + u_y^2 + v_x^2 + v_y^2)\, dx\, dy$$

The Euler equations for this problem yield

$$\lambda \Delta u = (uE_x + vE_y + E_t)E_x$$
$$\lambda \Delta v = (uE_x + vE_y + E_t)E_y$$

where E_x, E_y, and E_t are the derivatives of image brightness. What are the natural boundary conditions?

Problem 2: The formulation of the optical flow problem as above uses as a measure of "unsmoothness" the sum of squares of first partial derivatives of the components of the optical flow. Consider a sphere centered at $X = Y = 0$, of radius R, rotating about an axis parallel to the y-axis with angular velocity ω. Suppose the sphere is far away from the camera (in relation to its radius) and we approximate perspective projection by parallel projection (scaled orthographic projection) where

$$x/f = X/Z_0 \quad y/f = Y/Z_0$$

with (x, y) being the image coordinates corresponding to the point $(X, Y, Z)^T$, with f and Z_0 constant. Find the motion field components $u(x, y)$ and $v(x, y)$. Where does the measure of "unsmoothness" become large (or even singular)?

Problem 3: In the "height and gradient" approach to Shape from Shading, one minimizes

$$\iint \big(E(x,y) - R(p,q)\big)^2 + \lambda(p_x^2 + p_y^2 + q_x^2 + q_y^2) + \mu\big((z_x - p)^2 + (z_y - q)^2\big)\, dx\, dy$$

Noting that there are *three* unknown functions, find the three Euler equations for this problem.

Problem 4: In interpolating smooth surfaces from sparse data one can choose a solution that minimizes some measure of "unsmoothness" (the data could be height known only on contour lines, or spot heights from binocular stereo).

What is the partial differential equation that must be solved if the measure of "unsmoothness" is

$$\iint (z_x^2 + z_y^2)\, dx\, dy$$

(that is, the integral of the gradient squared)?

What is the partial differential equation that must be solved if the measure of "unsmoothness" is instead

$$\iint (z_{xx} + z_{yy})^2 \, dx \, dy$$

(that is, the integral of the Laplacian squared)?

Problem 5: In order to get good signal-to-noise ratio, digital video cameras typically integrate light falling on each picture cell from one frame to the next (as opposed to using an electronic shutter yielding only a short exposure). They also read the image out from top to bottom, row by row, and typically the time taken to read out all rows equals the frame time (as opposed to reading the image out rapidly and having a gap in data output between frames).

Since different rows are read out at different times, the resulting image is not consistent in time. Consider how this effect would affect estimation of image motion (u, v) — assumed here to be the same for all points in the image. Assume that all pixels in a row are read out at the same time, that there are n rows and m columns of pixels in the imaging device, that the frame has height H and width W, and that the frame rate is F (frames per second).

(a) What is the effect of the time-inconsistency on the estimation of image velocity when the image does not move from frame to frame?

(b) What is the effect of the time-inconsistency on the estimation of image velocity when the image moves strictly in the horizontal (x) direction?

(c) What is the effect of the inconsistency when the image moves strictly in the vertical (y) direction? Show that a pattern moving in the vertical direction with speed v is compressed in the vertical direction — its vertical extent is multiplied by $1/(1 + v/v_s)$. Express v_s in terms of H, the image height, and T, the frame time.

(d) Is there a vertical velocity for which the image appears to be constant in the y-direction (that is, brightness is a function of x only)?

(e) Suppose that the underlying time-varying image is $E(x, y, t)$. Show that the image sequence being read out can be treated as if it were sampled from

$$E'(x, y, t) = E(x, y, t + ax + by)$$

What are the values of the constants a and b? Modify the equations for finding image motion (u, v) (assumed constant over the image) to take into account the time-inconsistency.