6.972 Game Theory and Equilibrium Analysis

Homework 1

February 4, 2005

DUE: In lecture 4.

Reading Assignment: Chapters 1 and 2 of Fudenberg and Tirole.

Problem 1.1 Exercise 1.6 from Fudenberg and Tirole.

Problem 1.2 Exercise 1.7 from Fudenberg and Tirole

Problem 1.3 *(An auction)* An object is to be assigned to a player in the set $\{1, ..., n\}$ in exchange for a payment. Player i's valuation of the object is v_i, and $v_1 > v_2 > ... > v_n > 0$. The mechanism used to assign the object is a (sealed-bid) auction: the players simultaneously submit bids (nonnegative numbers), and the object is given to the player with the lowest index among those who submit the highest bid, in exchange for a payment.

In a *first price* auction the payment that the winner makes is the price that he bids.

Formulate a first price auction as a strategic game and analyze its Nash equilibria. In particular, show that in all equilibria player 1 obtains the object.

Problem 1.4 *(A war of attrition)* Two players are involved in a dispute over an object. The value of the object to player i is $v_i > 0$. Time is modeled as a continuous variable that starts at 0 and runs indefinitely. Each player chooses when to concede the object to the other player; if the first player to concede does so at time t, the other player obtains the object at that time. If both players concede simultaneously, the object is split equally between them, player i receiving a payoff of $v_i/2$. Time is valuable: until the first concession each player loses one unit of payoff per unit of time.

Formulate this situation as a strategic game and show that in all Nash equilibria one of the players concedes immediately.

Problem 1.5 Exercise 2.1 from Fudenberg and Tirole.

Problem 1.6 Exercise 2.2 from Fudenberg and Tirole.

Problem 1.7 Note that a function $u_i(\cdot)$ is *upper semi-continuous* at a, if, for any sequence a^n converging to a, $\limsup_{n\to\infty} u_i(a^n) \leq u_i(a)$. Note also that a function u_i has a *continuous maximum* if $u_i^*(a_{-i}) \equiv \max_{a_i} u_i(a_i, a_{-i})$ is continuous in a_{-i}.

Let A_i be a nonempty, convex, and compact subset of a finite-dimensional Euclidean space, for all i. Assume, for all i, u_i is quasiconcave in a_i, is upper semi-continuous in a, and has a continuous maximum. Show that there exists a pure strategy Nash equilibrium.