6.972 Game Theory and Equilibrium Analysis

Homework 5

April 8, 2005

DUE: In lecture 18

Problem 5.1 In class we have showed that repeated game can lead to worse outcomes than in the one shot game using the following example

P1 \ $P2$	A	B	C
A	$2,2$	$2,1$	$0,0$
B	$1,2$	$1,1$	$-1,0$
C	$0,0$	$0,-1$	$-1,-1$

For appropriate value of δ, this game has a SPE in which (B,B) is played at each period. Consider the following strategy profile

I. Play B is every period until someone deviates in which case go to II.

II. Play C; If no one deviates, go to I; If someone deviates, stay in II.

Check whether this is an SPE and prove it.

Problem 5.2 (**Fudenberg and Maskin**): *Let v^* be the set of feasible strictly individually rational payoffs. Assume that $\dim V^* = N$. Then for any $v \in V^*$, $\exists \underline{\delta} < 1$, $\forall \delta > \underline{\delta}$, there is an SPE of $G^\infty(\delta)$ with average payoff of v*

Consider the following strategy profile for the Fudenberg and Maskin's Folk Theorem we discussed in class.

I. Play a_i, $g(a) = v$, so long as no one deviates. If j alone deviates, go to II_j.

II_j. Play m_i^J for T periods. Go to III_j if no one deviates. If during phase II_j, k deviates from m_k^j, restart phase II_k.

III_j. Play a_i^j so long as no one deviates. If k deviates, go to II_k.

Check whether this is an SPE and prove it.

Problem 5.3 Consider the following game with three players. Player 1 chooses row; player 2 chooses column; and player 3 chooses matrix

Matrix A

P1 $\backslash P2$	L	R
U	$1,1,1$	$0,0,0$
D	$0,0,0$	$0,0,0$

Matrix B

P1 $\backslash P2$	L	R
U	$0,0,0$	$0,0,0$
D	$0,0,0$	$1,1,1$

5.3 (a) What is the minmax level?

5.3 (b) What is the set of feasible payoffs?

5.3 (c) For any $\delta \in (0,1)$, show that there is no SPE of G^∞ with average payoff less than $\frac{1}{4}$?

Problem 5.4 Consider the following strategic form game:

P1 / P2	U	M	D
U	$4,4$	$0,0$	$0,5$
M	$0,0$	$3,3$	$0,0$
D	$5,0$	$0,0$	$1,1$

1. How many pure strategies does each player have in this strategic game? Find all the Nash equilibria of the stage game (pure and mixed).

   Suppose this stage game is played twice. Players' payoffs are given by discounted average of stage payoffs, with discount factor $\delta \in (0,1)$. After the first round, all the actions are observed.

2. How many pure strategies does each player have in this finitely repeated game?

3. Show that if the discount factor, δ, in this game is close enough to 1, the following play (outcome path) can be supported as a subgame perfect equilibrium: [(U,U),(M,M)]. Find the corresponding strategies at this subgame perfect equilibrium.

4. Explain why twice repetition can ensure cooperation (i.e., play of Pareto efficient non-Nash equilibrium play in stage game) while this is not possible in finitely repeated prisoner's dilemma.

5. Now consider the T-period repeated play of this stage game. What is the outcome path of the subgame perfect equilibrium that yields the highest discounted average payoff for both players (i.e., the 'best SPE')? Describe the outcome path of the best SPE as T$\rightarrow \infty$ and $\delta \rightarrow 1$. Contrast this with the limiting behavior of the T-period repeated prisoner's dilemma. (Recall that $T \rightarrow \infty$ and infinitely repeated game are different.)