6.012 Electronic Devices and Circuits
Formula Sheet for Final Exam, Fall 2003

Parameter Values:

$q \;=\; 1.6x10^{\;\;19}\; Coul$

$_{o} = \; 8.854x10^{\;\;14}\; F/cm, \qquad _{r,Si} = \; 11.7, \qquad _{r,SiO_2} = \; 3.9$

$_{Si} \quad 10^{\;\;12}\; F/cm, \qquad _{SiO_2} \quad 3.5\,x10^{\;\;13}\; F/cm$

$n_i[Si@R.T] \quad 10^{10}\; cm^{\;\;3}$

$kT/q \quad 0.025\,V; \quad (kT/q)\ln 10 \quad 0.06\,V$

$1 \quad m = 1x10^{\;\;4}\; cm$

Periodic Table:

III	IV	V
B	C	N
Al	Si	P
Ga	Ge	As
In	Sn	Sb

Drift/Diffusion:

Drift velocity: $\bar{s}_x = \pm \quad _m E_x$

Conductivity: $\quad = q(\quad _e n + \quad _h p)$

Diffusion flux: $F_m = \quad D_m \dfrac{\partial C_m}{\partial x}$

Einstein relation: $\dfrac{D_m}{\;_m} = \dfrac{kT}{q}$

Electrostatics:

$$\frac{dE(x)}{dx} = \quad (x) \qquad E(x) = \frac{1}{\;} \quad (x)dx$$

$$\frac{d \quad (x)}{dx} = E(x) \qquad (x) = \quad E(x)dx$$

$$\frac{d^2 \quad (x)}{dx^2} = \quad (x) \qquad (x) = \quad \frac{1}{\;} \quad (x)dxdx$$

The Five Basic Equations:

Electron concentration: $\dfrac{\partial n(x,t)}{\partial t} \quad \dfrac{1}{q}\dfrac{\partial J_e(x,t)}{\partial x} = g_L(x,t) \quad \left[n(x,t)\cdot p(x,t) \quad n_i^2\right] r(T)$

Hole concentraton: $\dfrac{\partial p(x,t)}{\partial t} + \dfrac{1}{q}\dfrac{\partial J_h(x,t)}{\partial x} = g_L(x,t) \quad \left[n(x,t)\cdot p(x,t) \quad n_i^2\right] r(T)$

Electron current density: $J_e(x,t) = q \quad _e n(x,t)E(x,t) + qD_e \dfrac{\partial n(x,t)}{\partial x}$

Hole current density: $J_h(x,t) = q \quad _h p(x,t)E(x,t) \quad qD_h \dfrac{\partial p(x,t)}{\partial x}$

Poisson's equation: $\dfrac{\partial E(x,t)}{\partial x} = \dfrac{q}{\;}\left[p(x,t) \quad n(x,t) + N_d^+(x) \quad N_a(x)\right]$

Uniform doping, full ionization, TE

n - type, $N_d >> N_a$

$$n_o \quad N_d \quad N_a \equiv N_D, \qquad p_o = n_i^2/n_o, \qquad _n = \frac{kT}{q}\ln\frac{N_D}{n_i}$$

p - type, $N_a >> N_d$

$$p_o \quad N_a \quad N_d \equiv N_A, \qquad n_o = n_i^2/p_o, \qquad _p = \quad \frac{kT}{q}\ln\frac{N_A}{n_i}$$

Uniform optical excitation, uniform doping

$$n = n_o + n' \qquad p = p_o + p' \qquad n' = p' \qquad \frac{dn'}{dt} = g_l(t) \quad (p_o + n_o + n')n'r$$

Low level injection, $n', p' << p_o + n_o$: $\dfrac{dn'}{dt} + \dfrac{n'}{\;_{min}} = g_l(t)$ with $_{min} \quad (p_o r)^{\;\;1}$

Flow problems (uniformly doped quasineutral regions with quasi-static excitation and low level injection; p-type example):

Minority carrier excess: $$\frac{d^2 n'(x)}{dx^2} \quad \frac{n'(x)}{L_e^2} = \quad \frac{1}{D_e} g_L(x) \qquad L_e \equiv \sqrt{D_e \quad {}_e}$$

Minority carrier current density: $$J_e(x) \quad qD_e \frac{dn'(t)}{dx}$$

Majority carrier current density: $$J_h(x) = J_{Tot} \quad J_e(x)$$

Electric field: $$E_x(x) \quad \frac{1}{q \quad {}_h p_o} J_h(x) \quad \frac{D_h}{D_e} J_e(x)$$

Majority carrier excess: $$p'(x) \quad n'(x) + \frac{\ }{q}\frac{dE_x(x)}{dx}$$

Non-uniformly doped semiconductor sample in thermal equilibrium

$$\frac{d^2 \quad (x)}{dx^2} = \frac{q}{\ }\left\{n_i\left[e^{q \quad (x)/kT} \quad e^{\ q \quad (x)/kT}\right] \quad \left[N_d(x) \quad N_a(x)\right]\right\}$$

$$n_o(x) = n_i e^{q \quad (x)/kT}, \quad p_o(x) = n_i e^{\ q \quad (x)/kT}, \quad p_o(x) n_o(x) = n_i^2$$

Depletion approximation for abrupt p-n junction:

$$(x) = \begin{array}{ccc} 0 & \text{for} & x < \quad x_p \\ qN_{Ap} & \text{for} & x_p < x < 0 \\ qN_{Dn} & \text{for} & 0 < x < x_n \\ 0 & \text{for} & x_n < x \end{array}$$

$$N_{Ap} x_p = N_{Dn} x_n$$

$${}_b \equiv \quad {}_n \quad {}_p = \frac{kT}{q} \ln \frac{N_{Dn} N_{Ap}}{n_i^2}$$

$$w(v_{AB}) = \sqrt{\frac{2 \quad {}_{Si}\left(\quad {}_b \quad v_{AB}\right)}{q} \frac{\left(N_{Ap} + N_{Dn}\right)}{N_{Ap} N_{Dn}}} \qquad \left|E_{pk}\right| = \sqrt{\frac{2q\left(\quad {}_b \quad v_{AB}\right)}{\quad {}_{Si}} \frac{N_{Ap} N_{Dn}}{\left(N_{Ap} + N_{Dn}\right)}}$$

$$q_{DP}(v_{AB}) = \quad AqN_{Ap} x_p\left(v_{AB}\right) = \quad A\sqrt{2q \quad {}_{Si}\left(\quad {}_b \quad v_{AB}\right) \frac{N_{Ap} N_{Dn}}{\left(N_{Ap} + N_{Dn}\right)}}$$

Ideal p-n junction diode i-v relation:

$$n(\text{-}x_p) = \frac{n_i^2}{N_{Ap}} e^{qv_{AB}/kT}, \quad n'(\text{-}x_p) = \frac{n_i^2}{N_{Ap}}\left(e^{qv_{AB}/kT} \quad 1\right); \qquad p(x_n) = \frac{n_i^2}{N_{Dn}} e^{qv_{AB}/kT}, \quad p'(x_n) = \frac{n_i^2}{N_{Dn}}\left(e^{qv_{AB}/kT} \quad 1\right)$$

$$i_D = Aqn_i^2 \quad \frac{D_h}{N_{Dn} w_{n,eff}} + \frac{D_e}{N_{Ap} w_{p,eff}} \quad \left[e^{qv_{AB}/kT} - 1\right] \qquad w_{m,eff} = \begin{array}{ll} w_m \quad x_m & \text{if } L_m >> w_m \\ L_m \tanh\left[\left(w_m \quad x_m\right)/L_m\right] & \text{if } L_m \sim w_m \\ L_m & \text{if } L_m << w_m \end{array}$$

$$q_{QNR,p\text{-}side} = Aq {}_{-w_p}^{-x_p} n'(x)dx, \qquad q_{QNR,n\text{-}side} = Aq {}_{x_n}^{w_n} p'(x)dx, \qquad \text{Note: } p'(x) \quad n'(x) \text{ in QNRs}$$

Ebers-Moll Model for Bipolar Junction Transistor (BJT) characterisitics (npn example; no base width modulation):

$$i_E = i_{hE} + i_{eE} = \quad Aqn_i^2 \quad \frac{D_h}{N_{DE} w_{E,eff}} + \frac{D_e}{N_{AB} w_{B,eff}} \quad \left[e^{qV_{BE}/kT} \quad 1\right] \qquad {}_E \equiv \frac{i_{hE}}{i_{eE}} = \frac{D_h}{D_e} \cdot \frac{N_{AB}}{N_{DE}} \cdot \frac{w_{B,eff}}{w_{E,eff}}$$

$$i_C \quad i_{eE}(1 \quad {}_B) = \quad Aqn_i^2 \frac{D_e}{N_{AB} w_{B,eff}} \left[e^{qV_{BE}/kT} \quad 1\right](1 \quad {}_B) \qquad \text{with} \qquad {}_B \quad \frac{w_{B,eff}^2}{2D_{e \quad e}} = \frac{w_{B,eff}^2}{2L_e^2}$$

$$i_B \quad i_{hE} + {}_B i_{eE} = i_{eE}({}_E + {}_B) = \quad Aqn_i^2 \frac{D_e}{N_{AB} w_{B,eff}} \left[e^{qV_{BE}/kT} \quad 1\right]({}_E + {}_B) \qquad {}_F \equiv \frac{i_C}{i_B} = \frac{(1 \quad {}_B)}{({}_E + {}_B)}$$

Large Signal BJT Model in Forward Active Region (FAR) (npn; with base width mod):

$$i_B(v_{BE}, v_{CE}) = I_{BS}\left[e^{qv_{BE}/kT} \quad 1\right] \quad \text{with} \quad I_{BS} \equiv \frac{1}{({}_o + 1)} I_{ES} = \frac{1}{({}_o + 1)} Aqn_i^2 \quad \frac{D_{eB}}{w_{B,eff} N_{AB}} + \frac{D_{hE}}{w_{E,eff} N_{DE}}$$

$$i_C(v_{BE}, v_{CE}) = \quad {}_o[1 + \quad v_{CE}]\, i_B(v_{BE}, v_{CE}) = \quad {}_o I_{BS}\left[e^{qv_{BE}/kT} \quad 1\right][1 + \quad v_{CE}]$$

$$\text{Break-point model:} \quad v_{BE,on} \quad 0.6V, \quad v_{CE,sat} \quad 0.2V$$

MOS Capacitor:

$$\text{Flat-band voltage:} \quad V_{FB} \equiv v_{GB} \ \text{ at which} \qquad (0) = \quad {}_{p \ Si}$$

$$V_{FB} = \quad {}_{p \ Si} \quad {}_m$$

$$\text{Threshold voltage:} \quad V_T \equiv v_{GC} \ \text{ at which} \qquad (0) = \quad {}_{p \ Si} + v_{BC}$$

$$V_T(v_{BC}) = V_{FB} \quad 2 \quad {}_{p \ Si} + \frac{1}{C_{ox}^*}\left\{2 \quad {}_{Si} qN_A \left[\left|2 \quad {}_{p \ Si}\right| \quad v_{BC}\right]\right\}^{1/2} \qquad x_{DT}(v_{BC}) = \sqrt{\frac{2 \quad {}_{Si}\left[\left|2 \quad {}_{p \ Si}\right| \quad v_{BC}\right]}{qN_A}}$$

$$\text{Inversion layer sheet charge density:} \qquad q_N^* = \quad C_{ox}^*[v_{GC} \quad V_T(v_{BC})]$$

$$\text{Accumulation layer sheet charge density:} \qquad q_P^* = \quad C_{ox}^*[v_{GB} \quad V_{FB}(v_{BC})]$$

Gradual Channel Approx. for MOSFET characteristics (n-channel; no channel length mod.):

$$\text{Valid for } v_{BS} \quad 0, \text{ and } \ v_{DS} \ge 0:$$

$$i_G(v_{GS}, v_{DS}, v_{BS}) = 0 \quad \text{and} \quad i_B(v_{GS}, v_{DS}, v_{BS}) = 0$$

$$i_D(v_{GS}, v_{DS}, v_{BS}) = \begin{matrix} 0 & \text{for} & \frac{1}{\ }[v_{GS} \quad V_T(v_{BS})] < 0 < v_{DS} \\ \frac{1}{2}\frac{W}{L} \quad {}_e C_{ox}^*[v_{GS} \quad V_T(v_{BS})]^2 & \text{for} & 0 < \frac{1}{\ }[v_{GS} \quad V_T(v_{BS})] < v_{DS} \\ \frac{W}{L} \quad {}_e C_{ox}^* \quad v_{GS} \quad V_T(v_{BS}) \quad \frac{v_{DS}}{2} \quad v_{DS} & \text{for} & 0 < v_{DS} < \frac{1}{\ }[v_{GS} \quad V_T(v_{BS})] \end{matrix}$$

$$\text{with} \quad V_T(v_{BS}) \equiv V_{FB} \quad 2 \quad {}_{p \ Si} + \frac{1}{C_{ox}^*}\left\{2 \quad {}_{Si} qN_A \left[\left|2 \quad {}_{p \ Si}\right| \quad v_{BS}\right]\right\}^{1/2}$$

$$\equiv 1 + \frac{1}{C_{ox}^*} \quad \frac{{}_{Si} qN_A}{2\left[\left|2 \quad {}_{p \ Si}\right| \quad v_{BS}\right]}^{1/2} \qquad \mathrm{C}_{\mathrm{ox}}^* \equiv \frac{{}_{ox}}{t_{ox}}$$

Large Signal MOSFET Model in Saturation (FAR) (n-channel; with base width mod.):

$$i_G(v_{GS},v_{DS},v_{BS}) = 0 \qquad i_B(v_{GS},v_{DS},v_{BS}) \quad 0$$

$$i_D(v_{GS},v_{DS},v_{BS}) = \frac{K}{2}\left[v_{GS} \quad V_T(v_{BS})\right]^2[1+ \quad v_{DS}]$$

$$\text{with} \quad K \equiv \frac{W}{L} \; {}_e C_{ox}^* \quad \text{and} \quad V_T = V_{FB} \quad 2 \;{}_p\;{}_{Si} + \frac{1}{C_{ox}^*}\left\{2 \;{}_{Si} q N_A\left[\left|2 \;{}_p\;{}_{Si}\right| \quad v_{BS}\right]\right\}^{1/2}$$

Small signal linear equivalent circuits:

p-n Diode

$$g_d = \frac{q}{kT} I_{BS}\, e^{qV_{AB}/kT} \qquad \frac{qI_D}{kT}$$

$$C_d = C_{dp} + C_{df}, \text{ where } \quad C_{dp}(V_{AB}) = A\sqrt{\frac{q\;{}_{Si}N_{Ap}}{2(\;{}_b \quad V_{AB})}} \quad \text{and}$$

$$C_{df}(V_{AB}) = \frac{qI_D}{kT}\frac{\left[w_p \quad x_p\right]^2}{2D_e} = g_d \;{}_d \quad \text{with} \quad {}_d \equiv \frac{\left[w_p \quad x_p\right]^2}{2D_e}$$

BJT (in FAR)

$$g_m \quad \frac{q|I_C|}{kT} \qquad g \quad = \frac{g_m}{\;{}_o} \qquad g_o \quad |I_C| \quad \text{or} \quad \left|\frac{I_C}{V_A}\right|$$

$$\mathrm{C} \quad = g_m \;{}_b + \text{ B-E depletion capacitance, where } \quad {}_b = \frac{w_B^2}{2\,D_{\substack{minority\\ .in\ base}}}$$

$$C \quad = \text{ B-C depletion capacitance}$$

MOSFET (in saturation)

$$g_m \quad \sqrt{2K\,|I_D|} = \mathrm{K}|\mathrm{V_{GS}} - \mathrm{V_T}| = \frac{2I_D}{(\mathrm{V_{GS}} - \mathrm{V_T})} \qquad g_o \quad |I_D| \quad \text{or} \quad \left|\frac{I_D}{V_A}\right|$$

$$g_{mb} = \quad g_m = \quad \sqrt{2K\,|I_D|} \qquad \text{with} \qquad = \frac{1}{C_{ox}^*}\sqrt{\frac{{}_{Si}qN_A}{|q \;{}_p| \quad V_{BS}}}$$

$$C_{gs} = \frac{2}{3}W\,LC_{ox}^* \qquad C_{sb}, C_{gb}, C_{db}: \text{ depletion region capacitances}$$

$$C_{gd} = W\,C_{gd}^* \quad \text{where } C_{gd}^* \text{ is the gate-to-drain fringing and overlap capacitance per unit gate width}$$

Single transistor analog circuit building block stages

Bipolar:-based stages

	Voltage gain, A_v	Current gain, A_i	Input resistance, R_i	Output resistance, R_o
Common emitter	$\frac{g_m}{[g_o+g_l]}\ (=\ \ g_m r_l')$	$\frac{g_l}{[g_o+g_l]}$	r	$r_o\ \ \ =\frac{1}{g_o}$
Common base	$\frac{g_m}{[g_o+g_l]}\ (= g_m r_l')$	1	$\frac{r}{[\ \ +1]}$	$[\ \ +1]r_o$
Emitter follower	$\frac{[g_m+g\ \ ]}{[g_m+g\ \ +g_o+g_l]}\ \ 1$	$\frac{g_l}{[g_o+g_l]}$	$r\ \ +[\ \ +1]r_l'$	$\frac{r_t+r}{[\ \ +1]}$
Emitter degeneration (series feedback)	$-\frac{r_l}{R_F}$		r $+[\ \ +1]R_F$	r_o
Shunt feedback	$\frac{[g_m\ \ \ G_F]}{[g_o+G_F]}\ \ \ g_m R_F$	$\frac{g_l}{G_F}$	$\frac{1}{g\ \ +G_F[1\ \ \ A_v]}$	$r_o\|R_F\ \ =\frac{1}{[g_o+G_F]}$

MOSFET-based stages

	Voltage gain, A_v	Current gain, A_i	Input resistance, R_i	Output resistance, R_o
Common source	$g_m\, r_l'$	•	•	$r_o\ \ \ =\frac{1}{g_o}$
Common gate	$[g_m+g_{mb}]r_l'$	1	$\frac{1}{[g_m+g_{mb}]}$	$r_o\ \ 1+\frac{[g_m+g_{mb}+g_o]}{g_t}$
Source follower	$\frac{g_m}{[g_m+g_o+g_l]}\ \ \ 1$	•	•	$\frac{1}{[g_m+g_o+g_l]}\ \ \ \frac{1}{g_m}$
Source degeneration (series feedback)	$-\frac{r_l}{R_F}$	•	•	r_o
Shunt feedback	$\frac{[g_m\ \ \ G_F]}{[g_o+G_F]}\ \ \ g_m R_F$	$\frac{g_l}{G_F}$	$\frac{R_F}{G_F[1\ \ \ A_v]}$	$r_o\|R_F\ \ =\frac{1}{[g_o+G_F]}$

OCTC/SCTC

OCTC estimation of $_{\text{HI}}$: $\ _{HI}\ \ \ \ \underset{i}{[\ \ _i]^{1}}\ \ ^{1}=\ \ \underset{i}{R_iC_i}^{\ \ 1}$

with R_i defined as the equivalent resistance in parallel with C_i, with all other parasitic device C's (C 's, C 's, C_{gs}'s, C_{gd}'s, etc.) open circuited.

SCTC estimation of $_{LO}$: $\ _{LO}\geq\ \ \underset{j}{}\ \ _j=\ \ \underset{j}{}\ [R_jC_j]^{\ 1}$

with R_j defined as the equivalent resistance in parallel with C_j, with all other biasing and coupling C's (C_I's, C_O's, C_E's, C_S's, etc.) short circuited.

Difference- and Common-mode signals

Given two signals, v_1 and v_2, we can decompose them into two new signals, one (v_C) that is common to both v_1 and v_2, and one (v_D) that makes an equal, but opposite polarity, contribution to v_1 and v_2:

$$v_D \equiv v_1 \quad v_2 \text{ and } v_C \equiv \frac{[v_1 + v_2]}{2} \qquad v_1 = v_C + \frac{v_D}{2} \text{ and } v_2 = v_C \quad \frac{v_D}{2}$$

CMOS performance

Transfer characteristic

$$\text{In general}: \quad V_{LO} = 0, \quad V_{HI} = V_{DD}, \quad I_{ON} = 0, \quad I_{OFF} = 0$$

$$\text{Symmetry}: \quad V_M = \frac{V_{DD}}{2} \text{ and } N_{ML} = N_{MH} \qquad K_n = K_p \text{ and } \left|V_{Tp}\right| = V_{Tn}$$

$$\text{Minimum size gate}: \quad L_n = L_p = L_{\min}, \quad W_p = \left(\,_n / \,_p \right) W_n, \quad W_n = W_{\min}$$

Switching trimes and Gate delay

$$_{Charge} = \quad _{Discharge} \quad \frac{2V_{DD}C_L}{K_n \left[V_{DD} \quad V_{Tn}\right]^2}$$

$$C_L = n\left(W_n L_n + W_p L_p\right) C_{ox}^* = 3nW_{\min} L_{\min} C_{ox}^* \qquad \text{assumes} \quad _e = 2 \quad _h$$

$$_{Min.Cycle} = \quad _{Charge} + \quad _{Discharge} \quad \frac{12nL_{\min}^2 V_{DD}}{_e \left[V_{DD} \quad V_{Tn}\right]^2}$$

:Power disippation

$$P_{ave@Max.f} = C_L V_{DD}^2 f_{\max} \ \mu \ C_L V_{DD}^2 \Big/ \quad _{Min.Cycle} = \frac{_e W_{\min} C_{ox}^* V_{DD} \left[V_{DD} \quad V_{Tn}\right]^2}{L_{\min}}$$

$$PD_{ave@Max.f} = \frac{P_{ave@Max.f}}{\text{Inverter area}} \ \mu \ \frac{P_{ave@Max.f}}{W_{\min} L_{\min}} = \frac{_e \quad _{ox} V_{DD} \left[V_{DD} \quad V_{Tn}\right]^2}{t_{ox} \ L_{\min}^2}$$