John E. Parsons
Anna Obizhaeva

# Calisthenics Assignment Part #2. Copper Mine Valuation

The objective of this assignment is to implement a real options valuation of a mine. You should use the 10 step binomial tree method from Assignment #1. All parameters are the same: set the drift in the copper price to 10%, the annual volatility to 28%, the risk-free rate to 5%, and the convenience yield to zero and copper spot price to $2.65.

Output of a mine is fixed at 5,000,000 ounces per year. Forecasted annual cost is $12,500,000. Let us assume that a mine can not be abandoned.

(1) Spot and futures prices.
   a) Calculate the expected spot price at each horizon, based on information at t=0. What is the expected rate of growth in the security? How does this relate to the risk-free rate, and why.
   b) Calculate the futures price at each horizon, based on information at t=0. What is the expected rate of growth in the security? How does this relate to the risk-free rate, and why.

(2) Define a security which is a claim on the revenue stream of a mine (without cost deduction).

   a) First, assume some risk-adjusted discount rate and value this claim using DCF methodology.
   b) Second, value this claim again using futures prices for the revenue stream.
   c) Finally, value this claim using the binomial tree and risk-neutral probabilities. Do you answers in b) and c) match, and why?
   d) What is the correct risk-adjusted discount rate in (a) such that the valuation is the same for all three methods?

(3) What if you add forecasted costs? On the separate spreadsheet do the following calculations:
   a) First, value the mine by the risk adjusted discounting methodology. Use the correct risk-adjusted discount rate from 2(d).
   b) Second, value the mine again using futures prices. Do you have the same answer? Why or why not? Compare your methodology to the lecture notes calculation for the Midas Mine.
   c) Third, use the 10 step binomial tree to find value of a mine. Compare your results to (a) and (b).

(4) Now assume that the mine can be abandoned at no cost:

a) Calculate the nodes where the value of assets is negative and investors prefer to abandon this mine
b) Value a mine with abandonment option
c) Compare the value of a mine with no abandonment option and the one with abandonment option. Explain the difference

(5) Value a mine that can be temporary shut down and opened again, both at cost of $20,000:

a) What are the possible reasonable strategies?
b) Consider the copper prices contingent strategies, i.e. for a chosen threshold X, consider a strategy when a mine can be temporary closed down when copper price falls below X and opened when copper price increases above X. Choose the reasonable threshold X and value a mine. Explain your choice of X
c) Compare value of a mine with the one in previous cases. Explain the difference

(6) Now explore how different types of financing affect value of a mine. From now on assume that a mine can be abandoned as in (4) but there is no option of closing it down and opening later as in (5). Assume that the mine was financed with both debt and equity. The outstanding debt contract has annual annuity bond payments of 5,000,000 dollars due each period starting the first period (and skipping time 0):

a) Value the bonds at each point in time, going backward
b) Determine a new abandonment point for each period
c) Revalue the firm, the equity and the debt
d) How does debt financing effects the value of mine and why? To compare with (4) assume that the equity holders in (6) get the benefit of floating the debt either in the form of a direct cash payout or in the form of a lower capital investment at t=0. What is the value of equity in this option (6), inclusive of the avoided investment or the payout from the debt flotation?

(7) Assume now that a mine is financed with equity and with a copper-price linked debt contract. There are annual copper price contingent bond payments of 100,000 ounces of copper due each period starting the first period (and skipping time 0). Redo all of the calculations above.

a) Now set the new quantity of copper payment (use solver) so that the total value of the bond is the same as in case of the regular annuity
b) Demonstrate who benefits from floating the copper bond versus the regular annuity
c) What is the source of the higher value of mine?
d) Play around and find the maximum amount of money that can be raised on the copper bond, while keeping the asset value of the mine at least as high as under annuity bond.