24.400

Proseminar in philosophy I

Fall 2003

<u>Some *Grundlagen* easy-to-prove theorems</u>

(HP) $\forall F \forall G(\#F = \#G \leftrightarrow F \approx G)$

<u>Definitions</u>

'0' is defined as '$\#[x: x \neq x]$' (§74)

'1' is defined as '$\#[x: x=0]$' (§77)

$\ulcorner Snm \urcorner$ ($\ulcorner$n is the successor of m$\urcorner$) is defined as:

$\ulcorner \exists F \exists x(Fx \ \& \ \#F = n \ \& \ \#[y: Fy \ \& \ y \neq x] = m)\urcorner$ (§76)

$\ulcorner Zx \urcorner$ ($\ulcorner$x is a Number$\urcorner$) is defined as:

$\ulcorner \exists F(\#F = x)\urcorner$ (§72)

[$\ulcorner Nx \urcorner$ ($\ulcorner$x is a finite Number$\urcorner$) is defined as:

$\ulcorner \forall F((\forall x(Sx0 \supset Fx) \ \& \ \forall y(Fy \supset \forall z(Szy \supset Fz)) \supset Fx) \vee x = 0\urcorner$ (§83)]

<u>Theorems</u>

1. $\forall F(\#F = 0 \leftrightarrow \forall x{\sim}Fx)$ (§75)

Proof. Suppose $\#F = 0$, i.e. $\#F = \#[x: x \neq x]$. By (HP), $F \approx [x: x \neq x]$. By the definition of '$\approx$':

$\exists R(\forall x(Fx \supset \exists_1 y([x: x \neq x]y \ \& \ Rxy)) \ \& \ \forall y([x: x \neq x]y \supset \exists_1 x(Fx \ \& Rxy)))$

Since ${\sim}\exists y[x: x \neq x]y$, ${\sim}\exists_1 y([x: x \neq x]y \ \& \ Rxy)$, for any R, x, so $\forall x{\sim}Fx$. Similarly for the other direction.

2. $\forall x Sx0 \supset x = 1$ (§78, 1)

3. $\forall F(\#F = 1 \supset \exists x Fx)$ (§78, 2)

4. $\forall F(\#F = 1 \supset \forall x \forall y((Fx \ \& \ Fy) \supset x = y)$ (§78, 3)

5. $\forall F(\exists x Fx \ \& \ \forall x \forall y((Fx \ \& \ Fy) \supset x = y) \supset \#F = 1)$ (§78, 4)

6. $\forall x \forall y \forall x' \forall y' Sxy \ \& \ Sx'y' \supset (x = x' \leftrightarrow y = y')$ (§78, 5)

7. $\forall x(Zx \ \& \ x \neq 0 \supset \exists y(Zy \ \& \ Sxy))$ (§78, 6)